UNIT-I
PHYSICAL WORLD AND MEASUREMENT

CHAPTER-1
PHYSICAL WORLD

TOPIC-1
Physical Science

Revision Notes

- **Scope of Physics :**
  The scope of Physics is wide, covering a tremendous range of magnitude of physical quantities.
  Physics covers larger quantities as well as smaller quantities. For example,

(i) Life span of most unstable particle to life of an average star *i.e.*, from 10^{-22} to 10^{18} s on time scale.
(ii) Celestial Body's mass to electron's mass *i.e.*, from 10^{55} kg to 10^{-30} kg on mass scales.

- **Excitement in Physics :**
  Few reasons that makes physics exciting—

(i) It is very challenging and exciting to unfold and detect secrets of nature with imaginative new experiments.
(ii) There is an innovative approach for applying physical laws to many new inventions *i.e.*, projects, machines, etc.
(iii) Few basic concepts, fundamental principles and laws can explain wide range of physical quantities, complicated phenomena of nature, etc.

Know the Terms

- **Science** is a systematic attempt to understand a natural phenomenon in as much detail and depth as possible and use the knowledge so gained to predict, modify and control phenomenon.
- **Physics** is the study of the basic laws of nature and their manifestation in different phenomenon.
- **Mechanics :** It is an area of science concerned with behaviour of physical bodies when subjected to forces or displacements and subsequent effect of bodies on environment. It can also be defined as branch of science which deals with motion and forces on objects.
- **Electrodynamics :** It deals with rapidly changing electric and magnetic fields.
- **Optics :** It involves the behaviour and properties of light including its interactions with matter and construction of instruments that use or detect it.
- **Thermodynamics :** It is concerned with heat and temperature and their relation to energy and work.
- **Mesoscopic Physics :** It is the sub-discipline of condensed matter physics which deals with materials of an intermediate length scale between size of quantity of atoms and of materials measuring micro-metres.

TOPIC-2
Physical Laws: Nature and Forces

Revision Notes

- **Nature of Physical Laws :**
  Nature of physical laws can be explained on the basis of certain laws. Certain quantities remain same but several quantities may change with time. Those quantities which remain constant known as conserved quantities and this concept called is law of conservation.

- **Laws of Conservation of Physical Quantities :**
  There are four laws of conservation in classical physics.

(a) **Law of Conservation of Energy :** It states that energy can neither be created nor destroyed but it can be changed from one form to another *i.e.* total sum of all types of energy in the whole universe remains constant.

(b) **Law of Conservation of Mass :** It states that matter can neither be created nor can be destroyed. Einstein's theory of relativity modified this statement to $E = mc^2$ where m is mass and c is the speed of light in vacuum. According to this theory mass and energy are inter-convertible.

(c) **Law of Conservation of Momentum :** It is classified into two laws—

(i) **Law of Conservation of Linear Momentum :** It states that if no external force acts on a system, then its linear momentum remains constant.

(ii) **Law of Conservation of Angular Momentum :** It states that if no external torque acts on a system, then its angular momentum remains constant.

(d) **Law of Conservation of Charge :** It states that charge can neither be created nor destroyed. It can be transferred from one body to another. It is the basic law of conservation in nature.

Know the Terms

- **Law** is precise and summarised statement related to duly verified and authentic observations of natural phenomena eg. Newton's laws of motion etc.
- **Conserved Quantities** are those physical quantities which remain constant in a process *i.e.* total energy, total momentum, etc.
- **Technology** is defined as the study of new techniques of producing machines, gadgets, etc. by using scientific discoveries and advancements. It is application of physics.

CHAPTER-2
UNITS AND MEASUREMENTS

TOPIC-1
Units System and Measurement

Revision Notes

- **Units :** It is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.
- **Fundamental Units :** The physical quantities which are independent of each other and which can represent remaining physical quantities are called fundamental physical quantities and their units are called fundamental units. They are seven in number as mentioned below :
  Seven Fundamental physical quantities in SI system of units are :

(a) Mass - *kg* (Kilogram)
(b) Length - *m* (Meter)
(c) Time - *s* (Second)
(d) Temperature - K (Kelvin)
(e) Electric current - A (Ampere)
(f) Luminous Intensity - cd (Candela)
(g) Quantity of Matter - *mol* (Mole)

(ii) **Derived Units :** These are the units of measurement of all other physical quantities which can be obtained from fundamental units, e.g. Velocity - (m/s), Acceleration - m/s^2, Pressure - Pa, Force - N and so on.

- **System of Units :**

(a) **F. P. S. system -** Foot, Pound, Second.
(b) **C. G. S. system -** Centimetre, Gram, Second.
(c) **M. K. S. system -** Meter, Kilogram, Second.

- **Length Measurement :**

(a) **Direct Methods :**
  (i) The use of a metre scale for distances from $10^{-3}\,m$ to $10^{2}\,m$.

(ii) The use of a vernier callipers for distances upto $10^{-4}\,m$.
(iii) The use of a screw gauge and a spherometer for distances upto $10^{-5}\,m$.
(iv) Echo method used in radar and sonar.

(b) Indirect Methods :
(i) Parallax method.
(ii) Astronomical Telescope for size of an astronomical object.
(iii) Tunneling microscope for size of molecule.
(iv) Echo method used in radar and sonar.

- **Mass Measurement :**

(a) For large masses - Gravitational methods.
(b) For small masses - Mass spectrograph.

- **Time-intervals Measurement**

(a) Electric oscillators
(b) Electronic oscillators
(c) Solar clock
(d) Quartz crystal clock
(e) Atomic clock
(f) Decay of elementary particles using photographic emulsion techniques.
(g) Radio-active dating technique.

Know the Terms

- **Mass of a body** is defined as the quantity of matter in a body which can never be zero.
- **Length of an object** may be defined as the distance of separation between any two points at the extreme ends of the object.

Know the Formulae

- 1 AU = 1.496×10^{11} m.
- 1 ly = 9.46×10^{15} m.
- 1 par sec = 3.1×10^{16} m.
- 1 Å = 10^{-10} m; 1 nm = 10^{-9} m
  1 μm = 10^{-6} m, 1 mm = 10^{-3} m
- 60 seconds (of arc) = 1 min (arc)
- 60 min. (of arc) = 1 degree (of arc)
- 180 degree (of arc) = π radian
- Indirect methods for long and small distances :

  Angular diameter $(\theta) = \dfrac{d}{\text{D}}$

  d = diameter, D = distance, radius = r
- **Time Measurement :**

  $$\text{Fractional error} = \frac{\text{Difference in time}}{\text{Time}}$$
- **Magnification :**

(a) $\text{Linear Magnification} = \dfrac{\text{Size of image}}{\text{Size of object}}$

(b) $\text{Linear Magnification} = \sqrt{\text{Areal Magnification}}$

TOPIC-2
Dimensional Analysis and Error

Revision Notes

- **Use of Dimensional Equations :**

1. Conversion of one system of units into another.

2. Checking the accuracy of various formulae.
3. Derivation of formulae.

➢ **Error :**
The difference in the true value and the measured value of a quantity is called error of measurement.

➢ **Types of Errors :**
(a) Systematic Error
(b) Random Error
(c) Gross Error.

Know the Terms

➢ **Dimensions of physical quantity** are the powers to which the symbols of fundamental quantities are raised to represent a derived unit of that quantity.

➢ **Dimensional formula of the given physical quantity** is the expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity.

➢ **Dimensional constants** are the quantities whose values are constant and they possess dimensions *e.g.* universal gravitational constant G etc.

➢ **Dimensional variables** are the quantities whose values are variable and they possess dimensions *e.g.* area, volume, etc.

➢ **Dimensional less constants** are the quantities whose value are constant but they do not possess dimensions *e.g.* mathematical constants– π, e and numbers.

➢ **Dimensional less variables** are the quantities whose values are variable and they do not have dimensions *e.g.* angle, strain, etc.

➢ **Accuracy** is a measure of how close the measured value is to true value of quantity.

➢ **Precision** describes the limitation of a measuring instrument.

Know the Formulae

➢ Conversion of one system of units into another

$$n_2 = \frac{n_1 u_1}{u_2} = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

➢ Mean absolute error

$$\Delta a_{mean} = \frac{1}{n} \times \sum_{i=1}^{i=n} |\Delta a_i|$$

➢ Fractional or Relative error

$$\delta a = \frac{\text{Mean absolute error}}{\text{True value}}$$

$$= \pm \frac{\Delta a_{mean}}{a_m}$$

$$a_m = \text{arithmetic mean}$$

➢ Percentage error

$$\delta a = \pm \frac{\Delta a_{mean}}{a_m} \times 100\%$$

➢ If $x = a + b$, $\quad \Delta x = \pm (\Delta a + \Delta b)$

➢ If $x = a - b$ $\quad \Delta x = \pm (\Delta a + \Delta b)$

➢ If $x = a \times b$, $\quad \frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

➢ If $x = \frac{a}{b}$, $\quad \frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

➢ If $x = a^n$, $\quad \frac{\Delta x}{x} = \pm n \left(\frac{\Delta a}{a}\right)$

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UNIT-II
KINEMATICS

CHAPTER-3
MOTION IN A STRAIGHT LINE

TOPIC-1
Motion & Velocity

Revision Notes

- **Rest :** An object or a particle is said to be in the state of rest when it does not change its position with time w.r.t. same reference point.

  Depending upon the position of observer, the state of rest of a particle is of two types :

(a) Absolute state of rest,

(b) Relative state of rest.

- **Motion :** An object or a particle is said to be in the state of motion when it changes its position with time w.r.t. same reference point.

  The motion of an object can be either linear or curvilinear, circular or in a plane or in a space.

(a) **Linear motion or Rectilinear or Translatory motion :**

(i) It is the motion in which a particle moves along a straight line with respect to a point of reference.

(ii) A body is said to be in linear motion if every constituent particle of the body move along parallel straight line and covers same distance in the given time.

(b) **Circular or Rotatory Motion :**

(i) A motion in which a particle or a point mass body is moving in a circle.

(ii) In rotatory motion all its constituent particles move simultaneously along concentric circles.

(c) **Oscillatory or Vibratory Motion :**

(i) In oscillatory motion the body moves back & forth repeatedly in definite interval of time about a fixed point.

(ii) If the amplitude of oscillating body is very small, the motion is called vibratory motion.

- **Dimensional Motion**

(a) **Motion in 1–D :**

(i) It is that motion in which a particle moves in one particular direction with respect to a point of reference.

(ii) In 1–D, the particle or a body moves along a straight line or a well defined path. Therefore, one dimensional motion is sometimes known as rectilinear or linear motion.

(b) **Motion in 2–D**

(i) If two out of three coordinates specifying the position of the object change with respect to time, the motion is called 2–D motion.

(c) **Motion in 3–D**

(i) If all the three coordinates specifying the position of the object change with respect to time, the motion is called 3–D motion.

- **Frame of Reference :** It is a system of co-ordinate axes attached to an observer having a clock with him, w.r.t. which the observer can describe position, displacement, acceleration, etc. of moving object. It is of two types :

(a) **Inertial frame**—It obeys Newton's first law.

(b) **Non-inertial frame**—It does not obey Newton's first law.

- **Path Length or Distance :**

(a) Length is defined as the actual path traversed by body during motion in a given interval of time.

(b) Distance is a scalar quantity.

(c) The S.I. unit of distance is metre and C.G.S. unit is centimeter.

(d) The value of distance traversed by a moving body can never be zero or negative.

- **Displacement :**

(a) Displacement of a body in a given time is defined as the change in the position of the body in a particular direction during that time. It may also defined as the shortest distance between initial and final position of the object.

(b) Displacement is a vector quantity as it possesses both magnitude and direction.

(c) Displacement of a body in a given time is represented by a vector drawn from the initial position to its final position.

(d) The unit of displacement is same as that of length.

(e) The value of displacement can be positive, zero or negative.

(f) The value of displacement can never be greater than the distance travelled.

(g) When a moving body returns to its starting point, then its effective displacement is zero.

➢ **Difference between Distance & Displacement :**

S. No.	Distance	Displacement
1.	Actual path traversed by object in given time.	Shortest distance between initial & final positions of object in given time.
2.	Scalar quantity.	Vector quantity.
3.	It cannot be zero or negative, it will be always positive.	It can be positive, negative or zero.
4.	It is either equal or greater than displacement but never less than displacement.	It is either equal or less than distance but never greater than distance.
5.	It can have many values depending upon path followed between two positions.	It has unique value.
6.	Between two positions of an object, it tells type of path followed.	It does not tell type of path followed.

➢ **Speed :** It is the ratio of total path length & corresponding time taken by an object.

(a) Speed is a scalar quantity.

(b) The speed of a body can be zero or positive but never negative.

(c) The speed of a body can increase or decrease with time.

(d) The C.G.S. unit of speed is cm/s and S.I. unit is m/s.

(e) If the speed of a body is zero, the body is at rest.

(f) The distance time graph of a body at rest, is a straight line parallel to time axis.

➢ **Types of Speed :**

(i) Uniform Speed. (Dimensions are [M^0LT^{-1}]) :

(a) A body is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.

(b) If a body moves with a constant speed, distance travelled by it in each second is the same.

The distance travelled at the end of 1st, 2nd, 3rd, ... second will be in the ratio 1 : 2 : 3 : .....

(ii) Variable Speed : A body is said to be moving with a variable speed if it covers equal distance in unequal intervals of time or unequal distances in equal intervals of time, how soever small these intervals may be.

(iii) Average or Mean Speed : If a body is moving with a variable speed, then the average speed of the body is defined as the ratio of total distance travelled by the body to the total time taken, *i.e.*,

$$\text{Average speed} = \frac{\text{Total distance traversed}}{\text{Total time taken}}$$

(iv) Instantaneous Speed : When a body is moving with a variable speed, then the speed of the body at a given instant of time is known as its instantaneous speed. The body possesses different speed at different instant.

➢ **Velocity :** It is as the ratio of displacement & the corresponding time interval taken by object.

(a) It is vector quantity.

(b) It can be positive, negative or zero.

(c) Unit—cm/s in C.G.S. system & m/s in SI.

(d) Dimensions—[LT^{-1}]

(i) Uniform velocity : A body is said to be moving with a uniform velocity, if it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.

(ii) Variable velocity :

(a) The body is said to be moving with a variable velocity if it covers equal displacements in unequal intervals of time or unequal displacements in equal intervals of time or changes direction of motion while moving with a constant speed.
(b) The position-time graph of a body moving with variable velocity is a curve.

(iii) Average or Mean velocity :

(a) When a body is moving with a variable velocity, then the average velocity of the body for a given time is defined as the ratio of the total displacement of the body to the total time taken, *i.e.*,

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

(b) When a body is moving with a uniform velocity, its average velocity is equal to its uniform velocity.

(iv) Instantaneous velocity : If a body is moving with a variable velocity, then the velocity of the body at a given instant of time is called its instantaneous velocity.

Know the Terms

- **Mechanics** deals with study of motion of material objects, with respect to the time.
- **Statics** deals with study of material objects at rest, with respect to the time.
- **Kinematics** deals with study of motion of material objects without taking into account the factors like nature of force, nature of bodies, etc. with respect to the time.
- **Dynamics** deals with study of motion of objects taking into account the factors which cause motion.
- **Uniform motion** is said to be in an object when velocity is uniform *i.e.* it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.
- **Non–uniform motion** is said to be in an object when it undergoes equal displacements in unequal intervals of time, howsoever small these intervals may be.

Know the Formulae

- Path length or distance, D = Speed × Time
- Displacement = Velocity × Time
- $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$
- $\text{Velocity} = \dfrac{\text{Displacement}}{\text{Time}}$
- Relative velocity —
  - $\vec{v}_{BA} = \vec{v}_A - \vec{v}_B$
  - $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$

TOPIC-2
Uniformly Accelerated Motion

Revision Notes

- **Accelerated motion :** When an object is moving in non-uniform motion, the velocity is different at different instants *i.e.* the velocity keeps on changing with time. This motion is an accelerated motion.
- **Acceleration :** It is defined as the ratio of change in velocity & the corresponding time taken by the mirror object *i.e.*

(a) It is vector quantity.
(b) It is either positive or negative.
(c) Negative acceleration is called retardation.
(d) Unit-m/s^2 in SI & cm/s^2 in CGS system.
(e) Dimensional formula– [LT^{-2}].

(i) Uniform acceleration : An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

(ii) Variable acceleration :

(a) An object is said to be moving with a variable acceleration when its velocity changes by unequal amounts in equal intervals of time.

(b) The velocity time graph of a body having variable acceleration is represented by a curve.

(iii) Average acceleration : When an object is moving with a variable acceleration, then the average acceleration of the body is defined as the ratio of the total change in velocity during motion to the total time taken, *i.e.*

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

(iv) Instantaneous acceleration :

(a) If a body is moving with a variable acceleration, then the acceleration of a body at the given instant of time is called instantaneous acceleration.

(b) If at an instant t, a body while moving with a variable acceleration shows a change in velocity $\Delta \vec{v}$ in a small interval of time Δt, so that $\Delta t \to 0$, then

$$\text{Instantaneous acceleration} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

where, $\frac{d\vec{v}}{dt}$ is the derivative of velocity $(\vec{v})$ w.r.t. time.

Know the Terms

- **Total displacement of the body** in the given time is equal to the area which velocity time-graph encloses with time axis.
- **Uniformly accelerated object** in a given time-interval is represented by the slope on the velocity-time graph for a given time-interval.
- **Acceleration of object** is the slope of velocity-time graph of uniformly accelerated motion.
- **Reaction time** is that time which a person takes to observe, think & act.

Know the Formulae

- Suppose
  u = initial velocity of body,
  a = uniform acceleration of the body,
  v = velocity of the body after time t,
  s = distance travelled by body in time t,
  s_n = distance travelled by body in n^{th} second.

(a) The equations of motion for accelerated body are :

(i) $v = u + at$ **(ii)** $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$ **(iv)** $s_n = u + \frac{a}{2}(2n - 1)$

(b) The equations of motion for retarded body (here, *a* is negative) are :

(i) $v = u - at$ **(ii)** $s = ut - \frac{1}{2}at^2$

(iii) $v^2 = u^2 - 2as$ **(iv)** $s_n = u - \frac{a}{2}(2n - 1)$

(c) The equations of motion for a body falling down under gravity (here, $a = +g$, $s = h$) are :

(i) $v = u + gt$ **(ii)** $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$ **(iv)** $h_n = u + \frac{g}{2}(2n - 1)$

(d) The equations of motion for a body going up under gravity (here $a = -g$, $s = h$) are :

(i) $v = u - gt$ **(ii)** $h = ut - \frac{1}{2}gt^2$

(iii) $v^2 = u^2 - 2gh$ **(iv)** $h_n = u - \frac{g}{2}(2n - 1)$

(e) The maximum height attained by a body thrown vertically upwards with initial velocity u is

$$h_{max} = \frac{u^2}{2g}$$

(f) Time taken to reach the maximum height is

$$t = \frac{u}{g}$$

(g) Total time taken by body in going up and coming down,

$$T = 2t = \frac{2u}{g}$$

(h) The initial velocity of body in order to attain height h is,

$$u = \sqrt{2gh}$$

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CHAPTER-4
MOTION IN A PLANE

TOPIC-1
Scalar and Vector quantities

Revision Notes

Scalar : A physical quantity which has only magnitude and no direction is called a scalar quantity or scalar.
Vector : A physical quantity which constitutes of magnitude as well as direction is called a vector quantity or vector. If it follows law of vector addition too.

➢ **Unit vector :**

(i) A unit vector of $\vec{A}$ is written as $\hat{A}$ and is given by $\hat{A} = \vec{A}/|A|$

(ii) A unit vector is dimensionless quantity of magnitude equal to unity. Its magnitude is 1 and it can have any direction.

(iii) In cartesian co-ordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x, y and z-axes, respectively.

➢ **Polar vectors :** These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.

➢ **Axial vectors or rotational vectors :** These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.

➢ **Some vector laws :**

(1) General law for addition of vector : It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with tail of third vector and so on then resultant vector is represented in magnitude and direction by the line starting from tail of first vector to head of last vector.

(2) Triangle Law : It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of triangle taken in opposite order.

(3) Parallelogram Law : It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

➢ **Lami's Theorem :** It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces, *i.e.*

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Know the Terms

➢ **Modulus of vector** is the magnitude of vector.

➢ **Equal vectors** are those vectors which have equal magnitude and same direction.

- **Negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of given vector.
- **Co-initial vectors** are those vectors which have common initial point.
- **Collinear vectors** are those vectors which are having equal or unequal magnitudes and are acting along parallel straight lines.
- **Coplanar vectors** are those vectors which are acting in the same plane.
- **Localised vector** is that vector whose initial point is fixed.
- **Non-Localised vector** is that vector whose initial point is not fixed.
- **Zero or Null vector** is that vector which has zero magnitude and an arbitrary direction and represented by $\vec{0}$.
- **Displacement vector** is that vector which tells how much and in which direction an object has changed its position in a given interval of time.
- **Resultant vector** is defined as that single vector which produces the same effect as is produced by two or more individual vectors together.
- **Equilibrant vector** is a single vector which balances two or more vectors acting on a body at the same time.

Know the Formulae

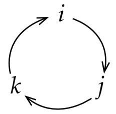
- $$\vec{R} = \vec{A} + \vec{B}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}\text{; } \beta = \text{angle of } \vec{R} \text{ with } \vec{A}.$$
- $$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$\tan\beta = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)} = \frac{B\sin\theta}{A - B\cos\theta}$$
- **Relative Velocity—** $$\vec{V}_{BA} = \vec{V}_A - \vec{V}_B$$
- $$\vec{A} = A_x\hat{i} + A_y\hat{j} \text{ and } A_x = A\cos\theta,\ A_y = A\sin\theta \text{ (In 2D)}$$
- $$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k},\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \text{ (In 3D)}$$
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2},\ |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$
- $$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$
- **Unit Vector of $\hat{A}$ is**
$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$
- $$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$
- If two vectors are parallel to each other *i.e.*, $\theta = 0°$
$$\vec{A}\cdot\vec{B} = AB\cos 0 = AB$$
$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$
- If two vectors are perpendicular to each other *i.e.*, $\theta = 90°$.
$$\vec{A}\cdot\vec{B} = AB\cos 90° = 0$$
$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$$
- If two vectors are parallel to each other *i.e.*, $\theta = 0$
- $$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$
$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$
- If two vectors are parallel to each other *i.e.*, $\theta = 0°$

$$\vec{A}\times\vec{B} = AB\sin 0° = 0$$
$$\hat{n} = \vec{0}$$

➢ If two vectors are perpendicular to each other *i.e.*, $\theta = 90°$

$$\therefore \quad \vec{A} \times \vec{B} = AB \sin 90° = AB$$

➢ Trick to remember Cross product



$$\hat{i}\times\hat{j} = \hat{k},$$
$$\hat{j}\times\hat{k} = \hat{i},$$
$$\hat{k}\times\hat{i} = \hat{j}$$

and
$$\hat{i}\times\hat{k} = -\hat{j},$$
$$\hat{k}\times\hat{j} = -\hat{i},$$
$$\hat{j}\times\hat{i} = -\hat{k}$$

➢ $$\text{Area of triangle} = \frac{1}{2}|\vec{A}\times\vec{B}|$$

➢ $$\text{Area of parallelogram} = |\vec{A}\times\vec{B}|$$

➢ $$\text{Unit vector} \perp \text{ to } \vec{A} \textit{ and } \vec{B}$$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

$$\text{where, } \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

➢ $$\text{If } \vec{A}+\vec{B}+\vec{C} = \vec{0}$$

then
$$\vec{A}\times\vec{B} = \vec{B}\times\vec{\ } = \vec{C}\times\vec{A}$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|A|\,|B|}$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A|\,|B|}$$

$$\tan\theta = \frac{\vec{A}\times\vec{B}}{\vec{A}\cdot\vec{B}}$$

TOPIC-2
Projectile Motion

Revision Notes

➢ **Projectile :** Projectile is defined as a body thrown with some initial velocity except vertically upward or downward and then allowed to move under the action of gravity alone, without being propelled by an engine or fuel or any source. The path followed by a projectile is known as its **trajectory**.

➢ **Centripetal force :**

(a) It is the force required to move the body in circular path with a constant angular velocity.

(b) The centripetal force acts on the particle along the radius which is directed towards the centre of circular path.

(c) The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path, therefore the work done by the centripetal force is zero.

(d) The centripetal force is provided in different ways, in different types of circular motions.

Know the Terms

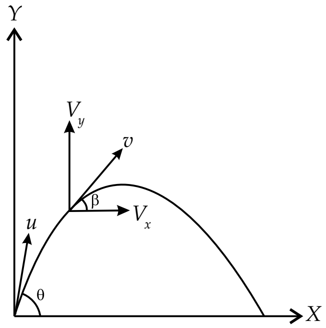
- **Angular displacement** of the object moving around a circular path is defined as the angle traced out by radius vector at the centre of circular path in given time. It is vector quantity.
- **Angular velocity** of an object in circular motion is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Uniform circular motion** is the motion when a point object is moving on a circular path with a constant speed.

Know the Formulae

- **For motion along *X*-axis,** $v_x = u_x + a_x t$ and $x = x_0 + u_x t + \frac{1}{2}a_x t^2$
- **For motion along *Y*-axis,** $v_y = u_y + a_y t$ and $y = y_0 + u_y t + \frac{1}{2}a_y t^2$
- **Velocity of projective at an instant of its flight is**

$$v = \sqrt{v_x^2 + v_y^2}$$

and $$\tan\beta = \frac{v_y}{v_x}$$



u = initial speed
θ = angle of projection

- **Angular projection of projectile :**

(i) Time of flight, $$T = \frac{2u\sin\theta}{g}$$

(ii) Maximum height, $$h = \frac{u^2\sin^2\theta}{2g}$$

(iii) Horizontal range, $$R = \frac{u^2\sin 2\theta}{g}$$

(iv) Maximum horizontal range $R_{max} = \frac{u^2}{g}$ for $\theta = 45°$

(v) Range is same for for angles θ and (90° – θ) if *u* & *g* remains unchanged

- **Circular Motion**
  - $\omega = \theta/t$
  - $v = l/t$
  - $w = 2\pi\nu = \frac{2\pi}{t}$
  - $a_c = w^2 r = wv = v^2/r$
  - $a_T = r\alpha$

where, a_c = Centripetal acceleration
a_T = Tangential acceleration
ω = angular velocity
ν = frequency
v = velocity
T = Time period

UNIT-III
LAWS OF MOTION

CHAPTER-5
LAWS OF MOTION

TOPIC-1
Newton's Laws of Motion

Revision Notes

- **Newton's I Law :** A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some non-zero external force to change the state. This law defines forces and is also called law of inertia.
- **Inertia** of a body is of three types as follows :

(i) **Inertia of rest** of a body is inability to change by itself, its state on its own.

(ii) **Inertia of motion** of a body is the inability to change by itself its state of uniform motion *i.e.* body in uniform motion can neither accelerate nor retard on its own and comes to rest.

(iii) **Inertia of direction** of a body is inability to change by itself its direction of motion, *i.e.*, body continues to move along the same straight line unless compelled by some external force to change it.

- **Linear momentum :** Linear momentum ($\vec{p}$) of a body is measured by the product of the mass (m) of the body and its velocity ($\vec{v}$) *i.e.*,

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity. Its direction is same as the direction of velocity of the body. The S.I. unit of linear momentum is $kgms^{-1}$.

- **Newton's II Law :** The rate of change of linear momentum ($\vec{p} = m\vec{v}$) of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force, *i.e.*,

$$\mathbf{F} \propto \frac{d\vec{p}}{dt}$$

or

$$\mathbf{F} = k\frac{d\vec{p}}{dt} = k\frac{d}{dt}(m\vec{v})$$

$$= km\left(\frac{d\vec{v}}{dt}\right) = km\vec{a}$$

where $\frac{d\vec{v}}{dt} = \vec{a}$, which is called acceleration of the body. Force can be defined in such a way that $k = 1$, then Newton's second law is written as

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

- **Newton's III Law :** To every action, there is always an equal and opposite reaction. The action and reaction act on different bodies, so they never cancel each other.

$$F_{AB} = -F_{BA}$$

F_{AB} = Force exerted on A by B

F_{BA} = Force exerted on B by A

Hence, $\vec{F}_{AB} + \vec{F}_{AB} = \vec{0}$

(i) **Principle of conservation of linear momentum :** From this principle, in an isolated system, the vector sum of the linear momentum of all the bodies of the system is conserved and is unaffected due to their mutual action and reaction. The total linear momentum of all the bodies in the system is given by

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = M\vec{v}_{c.m} = \text{constant}$$

where, M is that total mass of the system and $\vec{v}_{c.m}$ is the velocity of the centre of mass of the system.

(ii) Rocket propulsion : The propulsion of a rocket is based on the principle of conservation of linear momentum of Newton's third law of motion.

Suppose,

M_0 = Initial mass of rocket,

$\frac{\Delta M}{\Delta t}$ = Rate of ejection of fuel,

M = Mass of rocket at any instant,

$\vec{v}$ = Relative velocity of ejected gases w.r.t. rocket.

Then, thrust on the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \vec{v}$

Acceleration of the rocket in the absence of gravity $= \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M}$

Thrust on the rocket in the presence of gravity,

$$F = \frac{\Delta M}{\Delta t} \times \vec{v} - Mg$$

Acceleration of the rocket in the presence of gravity,

$$\vec{a} = \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M} - \vec{g}$$

Know the Terms

- **Force** is an external effort in the form of push or pull which can try to produce motion in a body at rest, or stops or try to stop a moving body or can change or try to change the direction of motion of the body.
- **Inertia** is the inherent property of a body, by virtue of which, the body doesn't change its state of rest or of uniform motion along a straight line, on its own. It depends upon the mass of the body.
- **Impulse :** When a large force acts on a body for a short time, then the measure of the total effect of force is called impulse of force. It can be found out

$$\text{Impulse} = \text{Force} \times \text{Time} = \vec{F}_{av} \times \Delta t$$

- **Inertial frame** is that in which the law of inertia is valid.
- **Non-inertial frame** is that in which the law of inertia is not valid.
- **Net force** is the vector sum of forces acting on an object.

Know the Formulae

- **Force Unit :** Newton in SI, dyne in cgs.
  Dimensional Formula : $F = [MLT^{-2}]$
- **Impulse Unit :** N-*s* in SI, dyne-*s* in cgs
  Dimensional Formula : $\vec{I} = [MLT^{-1}]$
- **Linear Momentum :** $\vec{p} = m\vec{v}$
- **Force** $= \frac{d\vec{p}}{dt} = m\vec{a}$
- **Impulse** = Force × time

$$\vec{I} = \vec{F} \times t = m(v - u)$$

- **Weight of person** = mg.
- **Principle of Conservation of Linear Momentum**

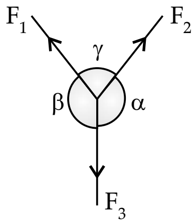
$$m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n = \text{Constant}$$

➢ **Recoil velocity of gun** $(\vec{v}_2) = \dfrac{-m_1 \vec{v}_1}{m_2}$

m_2 = Mass of gun
m_1 = Mass of bullet
v_1 = Velocity of bullet

➢ **Lami's Theorem :**
Three forces acting on body in equilibrium, then

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



TOPIC-2
Friction & Dynamics of Circular Motion

Revision Notes

➢ **Equilibrium of Concurrent forces :** Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.

(i) Conditions of equilibrium of concurrent forces : If $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, ... are the concurrent forces acting at the same point, then the point will be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = \vec{0}$$

➢ **Friction :** Friction is an opposing force which comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body. Frictional forces arise due to interlocking of irregularities present on the two surfaces which are in contact. From modern view, the frictional force arises due to strong atomic or molecular force of attraction between the two surfaces at the points of actual contact.

It is of two types :

(a) Internal friction : It arises on account of relative motion between every two layers of liquid. It is also known as viscosity of liquid.

(b) External friction : It arises when two bodies in contact with each other try to move or there is an actual relative motion between the two. It is also known as contact friction. Further it is of four types:

(i) Static friction is an opposing force which comes into play when one body tends to move over the surface of another body, but the actual motion has yet not started. It is a self-adjusting force.

(ii) Limiting friction is the maximum value of static friction. Limiting friction is the maximum opposing force that comes into play of one body is just at the average of moving over the surface of another body.

(iii) Dynamic or kinetic friction is the opposing force that acts between two surfaces in contact when one body is actually moving over the surface of another body.

(iv) Rolling friction is an opposing force that comes into play when one body is actually rolling over the surface of another body.

➢ **Laws of limiting friction :**

(a) The magnitude of the force of limiting friction (F) between two bodies in contact is directly proportional to the normal reaction (R) between them, *i.e.*, $F \propto R$.

(b) The direction of the force of limiting friction is always opposite to the direction in which one body is at the average of moving over the other.

(c) The force of limiting friction is independent of the apparent area of contact as long as normal reaction between the two bodies in contact remains the same.

(d) The force of limiting friction between any two bodies in contact depends on the nature of the surfaces in contact.

➢ **Coefficient of limiting friction** between two surfaces in contact (μ) is defined as the ratio of the force of limiting friction (F) and normal reaction (R) between them, *i.e.*,

$$\mu = F/R$$

The value of μ depends upon the nature of material and state of polish of two surfaces in contact.

Coefficient of kinetic friction, $$\mu_k = \frac{F_k}{R}$$

where, F_k is the force of kinetic friction and R is the normal reaction between the two surfaces in contact.

➢ **Advantages & Disadvantages of friction**

(a) Advantages :

(i) Walking and Brakes of vehicles will not be possible without friction.
(ii) No two bodies stick together without friction.
(iii) Writing on black board or paper will also be not possible.
(iv) Adhesives will lose their purpose.

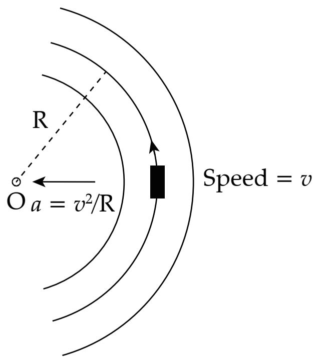
(b) Disadvantages :

(i) Friction always opposes the relative motion between any two bodies unnecessary expense of energy. It means output is always less than input.
(ii) Friction causes wear and tear of the parts of machinery in contact. Thus their life time reduces.
(iii) Frictional forces results in the production of heat, which causes damage to the machinery.

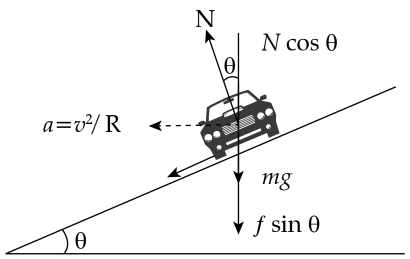
➢ **Ways of Reducing Friction.**

(i) By polishing
(ii) By lubrication
(iii) By proper selection of materials
(iv) By streamlining
(v) By using ball bearings.

➢ **Motion of car on a road.**



$$V_{max} = \sqrt{\mu_s R g}$$



$$V_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{1 - \mu_s \tan\theta}\right]$$

For $\mu_s = 0$ *i.e.*, for a frictionless banked road

$$V_{max} = \sqrt{rg\tan\theta}$$

where, μ_s is the coefficient of friction and R is the radius of the circle.

Know the Terms

- **Concurrent forces :** The forces which are acting at a point.
- **Contact forces** arises due to contact with some other object: solid or fluid.
- **Non-contact forces** are forces experienced by an object without actual contact. Eg. force of gravity of earth.
- **Centripetal force** is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.
- **Angle of friction** is the angle in which the direction of resultant of the force of friction and normal reaction makes with the direction of normal reaction. It is represented by θ.

$$\tan\theta = \mu$$

- **Angle of repose** is the maximum angle of inclination of a plane with the horizontal, at which the body placed on the plane is just in limiting equilibrium.
- **Centrifugal force** is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Know the Formulae

- **Angle of friction,** $\tan\theta = \mu.$
- **Angle of repose,** $\mu = \frac{F}{R} = \tan\theta.$
- **Centripetal force** $= \frac{mv^2}{r} = mr\omega^2.$

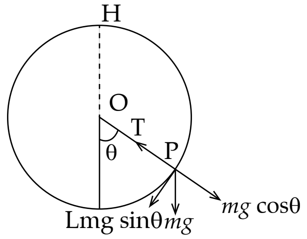
$$= mr(2\pi\nu)^2.$$

- $\tan\theta = v^2/rg.$
- $\tan\theta = \frac{h}{\sqrt{b^2 - h^2}} = v^2/rg.$

h = height between outer edge and inner edge

b = breadth of road

- $v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$
- **At any position of angular displacement θ along a vertical circle**



$$T = \frac{mv^2}{r} + mg\cos\theta.$$

- **At lowest point of vertical circle,** $\theta = 0°$; $T_L = \frac{mv_L^2}{r} + mg.$
- **At highest point of vertical circle,** $\theta = 180°$

$$T_H = \frac{mv_H^2}{r} - mg.$$

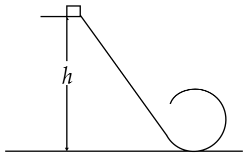
- **Minimum velocity at highest point, at H** $v_H = \sqrt{gr}.$

- **Minimum velocity at lowest point, at L** $v_L = \sqrt{5gr}$.
- **When,** $\theta = 90°$

$$v = \sqrt{3gr}.$$

- **Height through which a body should fall for looping the vertical loop**

$$h = 5r/2.$$



UNIT-IV
WORK ENERGY AND POWER

CHAPTER-6
WORK ENERGY AND POWER

TOPIC-1
Work and Power

Revision Notes

- **Work :** Work is done when the body is displaced actually through some distance in the direction of applied force.

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs\cos\theta$$

  - Work done is a scalar quantity. However, work done is positive when θ lies between 0 (zero) and $\pi/2$. Work done is negative when θ lies between $\pi/2$ and $3\pi/2$.
  - S. I. unit of work is joule (J)and the C.G.S unit of work is erg, where 1 joule = 10^7 erg.
  - Work done by a body does not depend on the time taken to complete the work.
- **Internal work or zero work.** The work in which muscles are strained, but work done is not useful, is called internal work. For example, when a person carrying load keeps on standing at the same place, work done is zero, but he gets tired on account of internal work.

  Dimensions : $[ML^2 T^{-2}]$

  Power : Power of a body is defined as the time rate of doing work by the body. Thus, in power, time taken by the body to complete the work is significant.

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

  - Here, θ is the angle between force $\vec{F}$ and velocity $\vec{v}$ of the body.
  - **Unit** : 1 W = 1 Js^{-1}
  - **Dimensions :** $[ML^2T^{-3}]$
  - Power is a scalar quantity.

Know the Terms

- **Conservative force** is a force if work done by or against the force in moving a body depends only on the initial & final positions of the body and not on the nature of path followed between initial and final positions, *e.g.*, gravitational force, electrostatic force between two electric charges, all central forces, etc.
- **Non-conservative force** is a force if work done by or against the force in moving a body from one position to another depends on the path followed between these two positions. *e.g.*, frictional forces, elastic forces, etc.

Know the Formulae

- Work= Force × Displacement in the direction of force
- $$W = \vec{F}.\vec{s}$$
  $$= Fs\cos\theta$$
- Maximum work
  When $\cos\theta = 0°$
  $W = Fs$
- Minimum Work when $\theta° = 90°$
  Then $W = Fs \times \cos 90° = 0$
- Work done by variable force :
  $$W = \int_{x_A}^{x_B} \vec{F}.d\vec{x}$$
- Power= $\dfrac{\text{Work done}}{\text{Time Taken}}$
- $$P = \vec{F}\cdot\vec{v}$$
- $$P = Fv\cos\theta.$$

TOPIC-2
Energy & Collision

Revision Notes

- **Energy :** Energy of a body is defined as the capacity of the body to do the work. Energy is a scalar quantity. It has the same units and dimensions as those of work. Some practical units of energy and their relation with S.I. unit of energy (joule) are :
  (i) 1 calorie = 4.2 J
  (ii) 1 kiloWatt hour (kWh) = 3.6×10^6 J
  (iii) 1 electron volt (1 eV) = 1.6×10^{-19} J
- **Work-Energy Theorem :** According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body and *i.e.*
  $$W = K_f - K_i$$
  K_f = final K.E.
  K_i = initial K.E.
- **Mechanical Energy Conservation :** Total mechanical energy of a system is conserved if the forces, doing work on it are conservative. It is also called principle of conservation of total mechanical energy.
  $$K + U = \text{constant}$$
  K = Kinetic energy
  U = Potential energy
- **Principle of Conservation of Energy :** This law states that energy can neither be created nor be destroyed, within an isolated system, *i.e.*, sum total of all kinds of energy in this universe remains constant. However, energy can be converted from one form to another, such that amount of energy disappearing in one form is always equal to amount of energy appearing in the other form.
  For example, when a body falls freely, its potential energy goes on decreasing and its kinetic energy goes on increasing. The total mechanical energy of falling body remains constant at all points.

- **Collisions :** When a body strikes against another body such that there is exchange of energy and linear momentum take place then the two are said to collide. Collisions are of two types :
  - **(i) Perfectly elastic collision** is that in which there is no change in kinetic energy of the system, *i.e.,*
    Total K.E. before collision = Total K.E. after collision.
    e.g., collisions between atomic and subatomic particles are perfectly elastic collisions.
  - **(ii) Perfectly inelastic collision** is that in which K.E. is not conserved. Here, the bodies stick together after impact. Linear momentum is conserved in every collision elastic as well as inelastic, further total energy is also conserved in all such collisions. Kinetic energy alone is not conserved in inelastic collisions.

Know the Terms

- **Kinetic Energy** is the energy possessed by the body by virtue of its motion. It is always positive.
- **Potential Energy** is the energy possessed by the body by virtue of its position. It can be both negative as well as positive.
- **Gravitational Potential Energy** is the energy possessed by the body by virtue of its position with respect to center of earth or other body.
- **Potential Energy of spring** is the energy associated with the state of compression or expansion of an elastic spring.
- **Heat Energy** is the energy possessed by a body by virtue of random motion of molecules of body.
- **Internal Energy** is the energy possessed by the body by virtue of particular configuration of its molecules.
- **Chemical Energy** comes from the molecules participating in the chemical reaction having different binding energies.
- **Nuclear Energy** is the energy obtained from conversion of nuclear mass into energy in the atomic nucleus. *i.e.,* Nuclear fission, Nuclear fusion.
- **Transformation of Energy** is the process of change of energy from one form to other.
- **Coefficient of Restitution or Coefficient of Resilience** is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by '*e*'.

Know the Formulae

- $$\text{Kinetic Energy (K.E.)} = \frac{1}{2}mv^2,$$
  where, m = mass, v = velocity of particles
- $$\text{Potential Energy (P.E.)} = mgh$$
- $$\text{Velocity } (v) = \sqrt{2gh}$$
- $$\text{Force in Spring (F)} = -kx,$$
  where, k = spring constant, x = compression.
- **Mass Energy Equivalence :**
  $$E = mc^2$$
  where, m = mass that disappears
  E = energy that appears
  c = velocity of light
- **Coefficient of Resilience** $$(e) = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$$
  $$e = \frac{v_2 - v_1}{u_1 - u_2}$$
  for perfectly elastic collision, $e = 1$
  for perfectly inelastic collision, $e = 0$
- **Elastic collision in 1- Dimension :**

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

- **Inelastic Collision in 1–Dimension :**

$$v = \frac{m_1u_1}{m_1 + m_2} \text{ if } u_2 = 0$$

$$\text{Loss in K.E.} = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$$

UNIT-V

MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

CHAPTER-7 SYSTEM OF PARTICLES AND ROTATIONAL MOTION

TOPIC-1

Centre of Mass & Motion of Rotational Particles

Revision Notes

- **Kinds of Motion of Rigid Body**

  (i)Pure Translational Motion : All the particles of body are moving together with same velocity at particular instant of time. eg. A car moving is a straight line.

  (ii)Pure Rotational Motion : A rigid body rotates about a fixed axis. Every particle of the body moves in a circle which lies in a plane perpendicular to axis and has its centre on the axis. eg. A Potter's wheel.

  (iii)Combination of Translational and Rotational Motion : The motion of rigid body, which is not pivoted or fixed in some way is either pure translation motion or a combination of translation and rotation. eg. A vehicle's wheel.

- **Center of Mass of a two particle system :** Position vector of centre of mass of a two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to sum of the products of masses of two particles and their respective position vectors.

- **Momentum Conservation :** Total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

  $\vec{p} = \mathrm{M}\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n$

  Differentiating it,

  $$\frac{d\vec{p}}{dt} = \mathrm{M}\frac{d\vec{v}}{dt} = \mathrm{M}\vec{a} = \vec{\mathrm{F}}_{ext}.$$

  This is Newton's II law.

  For isolated system, $\vec{\mathrm{F}}_{ext} = \vec{0}$.

  $$\therefore \quad \frac{d\vec{p}}{dt} = \mathrm{F}_{ext} = 0 \text{ or } \vec{p} = \text{constant}$$

  $$\therefore \quad \mathrm{M}\vec{v} = \text{Constant.}$$

- **Right handed Screw Rule :** It states that if right handed screw is placed with its axis perpendicular to plane containing two vectors is rotated from direction of $\vec{A}$ to direction $\vec{B}$ through smaller angle, then sense of advancement of tip of screw gives direction of $\left(\vec{A}\times\vec{B}\right)$ or $\vec{C}$.
- **Moment of Force or Torque :** Torque due to a force is moment of force and measures the turning effect to the force about the axis of rotation. The general expression for torque is

$$\vec{\tau} = \vec{r}\times\vec{F}$$

- **Angular Momentum and its Conservation :** Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about the axis. It is equal to the product of linear momentum of the particle and the perpendicular distance of the line of action of linear momentum from the axis of rotation. It is the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

$$\vec{L} = \vec{r}\times\vec{p} = rp\sin\phi$$
$$= \vec{d}\times\vec{p}$$

where, $d = r\sin\phi$ = perpendicular distance of line of action of $\vec{p}$ from the axis. Angular momentum is a vector quantity, whose direction is given by **right handed screw rule.**

- $\vec{L}\perp\vec{d}$ and $\vec{L}\perp\vec{p}$
- **Rate of change of angular momentum is torque**, *i.e.*, $\vec{\tau} = d\vec{L}/dt$.

As $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \vec{\tau}_{ext}$.

for isolated system $\vec{\tau}_{ext} = \vec{0}$.

$\therefore$ $\vec{\tau}_{ext} = \dfrac{d\vec{L}}{dt} = \vec{0}$.

So, $\vec{L}$ = constant.

- **Equilibrium of Rigid Bodies :**

1[st] Condition : A rigid body is said to be in translational equilibrium, if it remain at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all external forces acting on the body must be zero *i.e.*, $\Sigma\vec{F}_i = \vec{0}$.

Translational Static Equilibrium is of 3 types :

(i) Stable Equilibrium

(ii) Unstable Equilibrium

(iii) Neutral Equilibrium

2[nd] Condition : A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body must be zero *i.e.*, $\Sigma\vec{F}_i = \vec{0}$.

- **Principle of Moments**

(a) According to principle of moments, body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

Know the Terms

- **System** is formed when a collection of any number of particles interact with one another.
- **Internal forces** are all the forces exerted by various particles of the system on one another.
- **External forces** are those forces exerted on a given system by the agencies outside the system.
- **A rigid body** is defined as a system of particles, in which distance between any two particles does not change under the influence of external forces, where, size and shape of the body will remain unaffected under the effect of external forces.
- **The centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated. If all the forces acting on the body were applied on the centre of mass, the nature of motion of the body shall remain unaffected.
- **Angular velocity** of a particle is defined as the time rate of change of its angular displacement.
- **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- **Centre of gravity** of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.

Know the Formulae

- **Position vector of centre of mass of *n* particles system**

$$\vec{r} = \frac{\sum_{i=1}^{n} m_i \vec{r_i}}{M}, \text{where, M is the total mass to the system } i.e., M = \sum_{i=1}^{n} m_i$$

- **For two particle system**

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

- **Coordinates of centre of mass will be**

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

- **If centre of mass lies at origin *i.e., x = y = 0,***

$$\therefore \quad m_1 x_1 + m_2 x_2 = 0 \text{ and } m_1 y_1 + m_2 y_2 = 0$$

$$\therefore \; x_2 = \frac{-m_1 x_1}{m_2}, \; y_2 = \frac{-m_1 y_1}{m_2}$$

- **Velocity of C.M. of a system of two particles is**

$$\vec{v}_{CM} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2}}{m_1 + m_2}$$

Note : For units.
1. All masses in kg
2. All distances in metre
3. All velocity in m/s

- **Cross Product of two Vectors :**

(i) $$\vec{A} \times \vec{B} = \vec{C} = AB \sin\theta \; \hat{C}$$

$$\theta = \text{Angle between } \vec{A} \,\&\, \vec{B}$$

(ii) $$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

(iii) Unit Vector

$$\vec{C} = \frac{\vec{A} \times \vec{B}}{AB\sin\theta} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

(iv) $$|\vec{A} \times \vec{B}| = \text{Area of parallelogram.}$$

- **Angular Momentum**

$$\vec{L} = \vec{r} \times m\vec{v}$$

Vector form $$\vec{L} = \vec{r} \times \vec{p} = rp\sin\phi$$

m = kg, v = m/s , p = kg ms^{-1} , L = kgm^2/s

- **Equations of Rotational Motion :**

(a) $$\omega_2 = \omega_1 + \alpha t$$

(b) $$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

(c) $$\omega_2^2 - \omega_1^2 = 2\alpha\theta$$

(d) $v = r\omega$

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

(e) $a = r\alpha$

(f) Centripetal acceleration $= \dfrac{v^2}{r} = r\omega^2$

where symbols have their usual meanings

- **Torque :** $\vec{\tau} = \vec{r} \times \vec{F}$
  (i) Work done by torque $dW = \tau(d\theta)$
  (ii) Power of torque $P = \dfrac{dW}{dt} = \tau\left(\dfrac{d\theta}{dt}\right)$
  $P = \tau\omega$

TOPIC-2
Moment of Inertia & Radius of Gyration

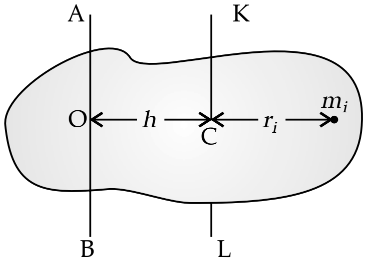
Revision Notes

- **Principle of Conservation of Angular Momentum :** According to this principle, when no external torque acts on a system of particles, then the total angular momentum of system always remains a constant. *i.e.*,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ... + \vec{L}_n = \text{constant.}$$

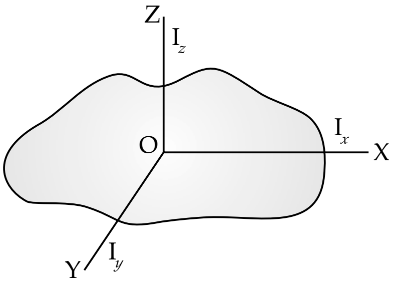
- **Theorem of Parallel Axes :** According to this theorem, moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through centre of mass C of the body in a direction parallel to AB, plus the product of total mass M of the body and square of the perpendicular distance between the two parallel axes. *i.e.*,

$$I_{AB} = I_{KL} + Mh^2$$



- **Theorem of Perpendicular Axes :** According to this theorem, the M.I. of plane lamina about any axis OZ perpendicular to the plane of the lamina is equal to the sum of moments of inertia of lamina about any two mutually perpendicular axes OX and OY in the plane of lamina, meeting at a point where the given axis OZ passes through the lamina. *i.e.*,

$$I_z = I_x + I_y$$



- **Laws of Rotational Motion**
  I Law : A body continues to be in a state of rest or in a state of uniform rotation about a given axis unless an external torque is applied on the body.

II Law : The rate of change of angular momentum of a body about a given axis is directly proportional to external torque applied on the body.

III Law : When a rigid body A exerts a torque on another rigid body B in contact with it, then the body B would exert an equal and opposite torque on the body A.

Know the Terms

- **Moment of inertia** of a body about a given axis is the property by virtue of which, the body opposes any change in its state of rest or state of uniform rotation about that axis. For a single particle, moment of inertia (I) is equal to product of mass (m) of the particle and square of perpendicular distance (r) of the particle from the axis of rotation.,

  i.e., $$I = mr^2.$$

  Moment of inertia is a scalar quantity, whose unit is kgm^2. It plays the same role in rotational motion as is played by the mass in linear motion.
- **Radius of gyration** of a body about a given axis is the distance (K) of a point from the given axis, where if whole mass of the body is concentrated, the body would have the same moment of inertia, as it has with the actual distribution of mass.
- **Kinetic energy of rotation** of a body is the energy possessed by body on account of its rotation about a given axis.

Know the Formulae

- **Moment of Inertia—**

  Unit—$kg.m^2$, **Dimension—**$[ML^2T^0]$

  (i) Circular ring— ($\perp^r$ to plane, at centre) $I = MR^2$

  (ii) Circular disc— ($\perp^r$ to plane, at centre) $I = \frac{1}{2}MR^2$

  (iii) Angular disc (or cylinder or ring) — (R = outer radius, r = inner radius) $I = \frac{1}{2}(R^2 + r^2)$

  (iv) Thin rod— (axis $\perp$ to its length at mid point) $I = \frac{Ml^2}{12}$, Where l is the length of the rod.

  (v) Soild cylinder— (along axis of cylinder) $I = \frac{1}{2}MR^2$

  (vi) Hollow cylinder— $I = MR^2$

  about its long axis of symmetry.

  (vii) Solid sphere— (about its diameter.) $I = \frac{2}{5}MR^2$

  (viii) Hollow sphere (or thin spherical shell) — (about its diameter) $I = \frac{2}{3}MR^2$

  (ix) Solid cylinder (or ring)— (about central axis) $I = M\left(\frac{l^2}{12}+\frac{R^2}{4}\right)$

  (x) (a) Solid cylinder (about axis through its CM)— $I = M\left(\frac{l^2}{12}+\frac{R^2}{4}\right)$

  (b) Hollow cylinder — $I = M\left(\frac{l^2}{12}+\frac{R^2}{2}\right)$

  about an axis passing through C.G. & $\perp$ to its own axis

  (xi) Uniform rectangular lamina (or thin slab)— about an axis passing through C.G. & $\perp$ to its plane.

  $$I = M\left(\frac{l^2+b^2}{12}\right)$$

  Where, l = length, b = breadth

(xii) Elliptical disc—

$$I = \frac{M}{4}\left(a^2 + b^2\right)$$

about an axis passing through its C.G. & $\perp$ to its plane.
(a = semi-major axis, b = semi-minor axis.)

(xiii) Uniform cone— about an axis joining the vertex to centre of its base.

$$I = \frac{3}{10}MR^2$$

(xiv) Triangular lamina—

$$I_1 = M \times h^2/6 \text{ (about the base as axis)}$$

$$I_2 = \frac{b^2}{6}.M \text{ (about the height as axis)}$$

$$I_3 = \frac{Mb^2h^2}{6\left(b^2+h^2\right)} \text{ (about the hypotenuse as axis)}$$

- **Kinetic energy of rotation—**

  Unit—Joule. $K.E. = \frac{1}{2}I\omega^2$,

- **Angular Momentum—** $L = I\omega$
- **Relation between Angular Momentum & Torque—**

$$\tau = I\alpha, \tau = dL/dt$$

$$L = kgm^2s^{-1}, \tau = N\text{–}m.$$

- **From Principle of Conservation of Angular momentum—**

$$\frac{I_1}{I_2} = \frac{\tau_1}{\tau_2}$$

- **Radius of Gyration—**

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ...r_n^2}{n}}$$

Here $r_1, r_2 ....r_n$ = perpendicular distance of particles from axis of rotation
n = total no. of particles.

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UNIT-VI
GRAVITATION

CHAPTER-8
GRAVITATION

TOPIC-1
Kepler's Laws, Universal Law of Gravitation, Acceleration Due to Gravity

Revision Notes

- **Kepler's Laws of Planetary Motion :**

  (a) Kepler's I Law (Law of Orbits) : Each planet revolves around the Sun in an elliptical orbit. The Sun is situated at one foci of the ellipse.

  (b) Kepler's II Law (Law of Areas) : The position vector of the planet from the Sun sweeps out equal area in equal interval of time. That is the areal velocity of the planet around the Sun is constant.

(c) Kepler's III Law (Law of Periods) : The square of the time period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

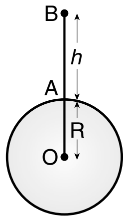
$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Universal Law of Gravitation : It states that every body in universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them and its direction is along line joining the two masses.

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = \frac{G m_1 m_2}{r^2}$$

- **Gravitational constant (G) :** It is defined as the force of attraction acting between two bodies each of unit mass, whose centres are placed at unit distance apart. The value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula of G = $[M^{-1}L^3T^{-2}]$. The value of G = 6.67×10^{-11} Nm^2kg^{-2}.
  The value of G being too small, we do not experience gravitational force our daily life.
- **Gravity :** It is the force of attraction exerted by Earth towards its centre on a body lying on or near the surface of the earth.
  Gravity is the measure of weight of the body.
  The weight of the body = mass (m) × acceleration due to gravity (g) = mg.
  The unit of weight of the body will be the same as those of force.
  Gravity is a vector quantity. It is always directed towards the centre of earth. Gravity holds the atmosphere around the earth.
- **Acceleration due to gravity (g) :** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone.
  Acceleration due to gravity is a vector quantity. It is directed towards the centre of Earth.
  The unit of g is ms^{-2} and its dimensional formula is $[M^0LT^{-2}]$.
  (a) The acceleration due to gravity does not depend upon (a) the mass of body, (b) shape or size of the body.
- **Variation of acceleration due to gravity.**

(a) Effect of altitude : $g' = \dfrac{gR^2}{(R+h)^2}$, h = height above earth surface.



when h is comparable with R and $h << R$.

$$g' = g\left(1 - \frac{2h}{R}\right)$$

From these relations, we conclude that acceleration due to gravity decreases with increase in height from the surface of earth.

(i) Fractional decrease in the value of g with height $= \dfrac{g - g'}{g} = \dfrac{2h}{R}$

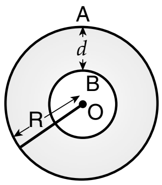
(ii) % decrease in the value of g $= \left(\dfrac{g-g'}{g}\right) \times 100 = \dfrac{2h}{R} \times 100\%$

(b) Effect of depth : $g' = g\left(1 - \dfrac{d}{R}\right)$, d = depth blow earth surface

(i) The acceleration due to gravity decreases with increase in depth d and becomes zero at the centre of the earth.

(ii) Decrease in the value of g with depth, $\Delta g = g - g'$

$$= \frac{gd}{R}$$



∴ Fractional decrease in the value of g with depth $= \dfrac{g-g'}{g} = \dfrac{d}{R}$

(iii) % decrease in the value of g with depth,

$$\frac{g-g'}{g} \times 100 = \frac{d}{R} \times 100\%$$

(c) Rotation of earth : $g = g' + R\omega^2 \cos^2 \lambda$

or $g' = g - R\omega^2 \cos^2 \lambda$

where, ω is the angular velocity of rotation of earth about its polar axis and λ is the latitude of a place.

(i) At the equator, $\lambda = 0°$,

so $g' = g - R\omega^2 \cos^2 0° = g - R\omega^2$

(ii) At the poles, $\lambda = 90°$,

so $g' = g - R\omega^2 \cos^2 90° = g$

Hence, the value of acceleration due to gravity increases from equator to pole due to rotation of earth. It means the value of g increases with latitude.

(iii) If a body of mass m is moved from equator to pole its weight increases by an amount

$$= m(g_p - g_e) = mR\omega^2.$$

(iv) Decrease in g in going from pole to equator is about 0·35%.

(v) When the earth stops rotating about its own axis, there will be no change in the value of g on the poles, but there will be increase in the value of g by about 0·35% at the equator.

(vi) When the earth starts rotating 17 times faster than its present rate, the value of g on the equator will become zero, but it will remain unchanged at the poles. In this situation, the duration of day will be 1 hour 24 minutes.

(d) Shape of the earth. Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. The polar radius of Earth is smaller than its equatorial radius by 21 km. As $g = GM/R^2$, so $g \propto 1/R^2$.

It means the value of acceleration due to gravity increases as we go from equator to pole. In fact, due to shape of the earth, at sea level, the value of acceleration due to gravity at pole is greater than at equator by 1·80 cms^{-2}.

Know the Terms

- **Areal velocity** may be defined as the area swept by the radius vector in unit time.
- **Cavendish method** determines the value of G.
- **Geodesic** is the shortest distance between two points on Earth.
- **Aphelion** is the nearest point of planet from Sun.
- **Perihelion** is the farthest point of planet from Sun.
- **Central forces** is a force that points from the particle directly towards a fixed point in space, the centre, and whose magnitude only depends on the distance of the object to the centre.
- **Riemann Geometry** is geometry of curve space.

Know the Formulae

- **Kepler's Law :**

$$T^2 = kr^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

Here, T = Time period and r = Semi major axis.

- **Newton's Gravitational Law :**

(a) $$F = \frac{Gm_1m_2}{r^2}$$

Units : F = Newton, m = kg, r = metre

(b) For system of mass bodies $\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \ldots + \vec{F}_{0n}$

- **Relation between g & G** $$g = \frac{GM}{R^2}$$

TOPIC-2
Gravitational Potential Energy & Satellites

Revision Notes

- **Gravitational potential energy (U) :** The amount of work done in bringing a body from infinity to that point.

$$U = \frac{GMm}{r}$$

- **Escape Velocity :** The minimum velocity with which a body must be projected up in the space, so as to enable it to just overcome the gravitational pull.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}. \qquad \text{as } g = \frac{GM}{R^2}$$

- **Satellite :** A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.
  - **(i) Natural satellites :** All those satellites were made by nature. e.g., Jupiter–16 moons. Saturn–18 moons.
  - **(ii) Artificial satellites :** All man-made satellites e.g., Aryabhatta, etc.
  - **(iii) Geostationary satellites :** A satellite which appears to be at a fixed position at a definite height to an observer on earth. It is also known as geosynchronous satellite.
- **Essential conditions for Geostationary satellites:**
  - **(a)** It should be at 36000 km above equator of earth.
  - **(b)** Its revolution period should be 24 hours about its axis.
  - **(c)** It should revolve in an orbit concentric and coplanar with equatorial plane.
  - **(d)** Its orbital speed is nearly 3.11 km/s and should be same sense of rotation as earth.
- **Polar satellite :** These satellites revolve in polar orbits around earth. A Polar orbit is that orbit whose angle of inclination with equatorial plane of earth is 90°.
- Gravitational forces are **central forces** as they act along the line joining the centres of the two bodies. The gravitational forces are **conservative forces.**

Know the Terms

- **Gravitational field** is the space around a material body in which its gravitational pull can be experienced.
- **Gravitational field intensity** of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.
- **Gravitational potential** at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.
- **Mass** of a body is the quantity of matter possessed by body.
- **Inertial mass of a body** is equal to the magnitude of external force required to produce unit acceleration in the body.
- **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.
- **Centre of Gravity (C.G.)** of a body placed in the gravitational field is that point where the net gravitational force of the field acts.

Know the Formulae

- **Gravitational Intensity :** $I = \frac{F}{m_0} = \frac{GM}{R^2} = g$

  Unit of I

  N/kg in SI, dyne/g in CGS Dimensions $[M^0LT^{-2}]$

- **Gravitational Potential :** $V = \frac{W}{m_0}$

$$V = -\frac{GM}{r}$$

Unit — J/kg in S.I. erg/g in C.G.S. system where W = Work done, m_0 = mass of body
Dimensions – $[M^0L^2T^{-2}]$.

- **Gravitational Potential Energy :** $U = \frac{-GMm}{r}$
- **Satellite :**

(a) Orbital speed : $v = R\sqrt{\frac{g}{R+h}} = \sqrt{\frac{GM}{R+h}}$

(b) Time period of revolution : $T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$

(c) Height of satellite : $h = \left(\frac{T^2R^2g}{4\pi^2}\right) - R$

- **Escape speed**

(i) $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$ $\quad \therefore \left[g = \frac{GM}{R^2}\right]$

(ii) Speed outside the gravitational field is

$$v' = \sqrt{v^2 - v_e^2}$$

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UNIT-VII
PROPERTIES OF BULK MATTER

CHAPTER-9
MECHANICAL PROPERTIES OF SOLIDS

TOPIC-1
Elastic Behaviour of Solids

Revision Notes

- **Stress** is defined as the restoring force acting per unit area of a deformed body, *i.e.*,

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

The S.I. unit of stress is N/m^2 and its dimensional formula = $[ML^{-1}T^{-2}]$. Stress is a tensor quantity. Normal Stress have following three types :

(a) Longitudinal stress : If a body changes its length under a deforming force and the stress is normal to the surface of the body then the stress is called longitudinal stress. The longitudinal stress can be a tensile stress or compression stress. The longitudinal stress produced because of increase in length of body under deforming force is known as **tensile stress.** The longitudinal stress produced due to decrease in length of body under a deforming force is known as **compression stress.**

(b) Volumetric or hydrostatic stress : If a body changes its volume under a normal deforming force acting on every surface of the body, the stress set up in the body is volumetric stress.

(c) Tangential stress : It is also called shearing stress. When a deforming force applied tangentially to the surface of the body changes the shape of the body without changing its volume, the stress set up is known as tangential stress. The shape of the body changes or the body gets twisted due to tangential stress.

- **Strain** is defined as the ratio of change in configuration of the body because of a deforming force on it, to the original configuration of the body it means

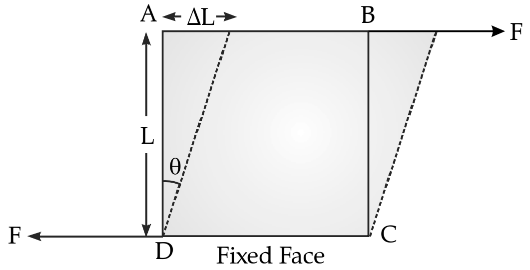
$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain can be of following three types, depending upon the direction of force applied :

(a) **Longitudinal strain** $= \dfrac{\text{change in length}}{\text{original length}} = \dfrac{\Delta l}{l}$

(b) **Volumetric strain** $= \dfrac{\text{change of volume}}{\text{original volume}} \quad \dfrac{\Delta V}{V}$

(c) Shearing strain is produced when the deforming force is applied parallel to the surface of a body and body changes its shape without changing its volume. Shearing strain is defined as the angle through which a vertical line perpendicular to the fixed surface gets rotated under the effect of a tangential deforming force.



Shearing strain is also defined as the ratio of displacement of a surface (ΔL) under the tangential deforming force to the perpendicular distance (L) of the displaced surface from the fixed surface, *i.e.*,

Shearing strain, $\theta = \dfrac{\Delta L}{L}$

Strain has no units and dimensions.

(a) If a beam is bent, both compression strain as well as extension strain are produced.

- Hooke's law states that within elastic limit, stress is directly proportional to strain, *i.e.*,
  Stress $\propto$ Strain.

Know the Terms

- **Interatomic forces** are those forces which are acting between the atoms due to electrostatic interaction between the charges of the atoms.
- **Intermolecular forces** are the forces which are acting between the molecules due to electrostatic interaction between the charges of the molecules.
- **Plasma state :** It is a state of matter in which the medium is in the form of positive and negative ions.
- **Crystalline solids** are those solids which have a definite external geometrical form and whose constituent atoms/ions/molecules are arranged in a definite pattern in three dimensions within the solid.
- **Amorphous solids** are those solids which have no definite external geometrical form and whose constituent atoms/ions/molecules are not arranged in a definite pattern in three dimensions within the solid.
- **Deforming force** is that force which when applied changes the configuration of the body.
- **Elasticity** is the property of the body by virtue of which the body regains its original configuration (length, volume or shape) when the deforming forces are removed.
- **Perfectly elastic body** is that body which perfectly regains its original form on removing the external deforming force from it, *e.g.*, quartz.
- **Plastic body** is that body which does not regain its original form at all on the removal of deforming force, howsoever small the deforming force may be, *e.g.*, putty and paraffin wax.
- **Elastic limit** is the upper limit of deforming force up to which if deforming force is removed, the body regains its original form completely and beyond which if the force is increased, the body loses its property of elasticity and it gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of material of a body.

Know the Formulae

- Normal stress (S) = F/A, $A = \pi r^2$
- Breaking force = Breaking stress × area of cross-section
- Longitudinal strain = $\dfrac{\Delta l}{l}$
- Volumetric strain = $\dfrac{\Delta V}{V}$
- Shearing strain, $\theta = \dfrac{\Delta L}{L}$

TOPIC-2
Modulus of Elasticity

Revision Notes

- **Modulus of elasticity or coefficient of elasticity (E)** of a body is defined as the ratio of stress to the corresponding strain produced, within the elastic limit, *i.e.,*

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of elasticity is of three types :

(a) Young's modulus of elasticity (Y) is defined as the ratio of normal stress to the longitudinal strain, within the elastic limit, *i.e.,*

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/A}{\Delta l / l} = \frac{F}{A} \times \frac{l}{\Delta l}$$

Y is the property of solid material only. Y increases on mixing the impurity in the solid and decreases on increasing the temperature of the solid body.

(b) Bulk modulus of elasticity (K) is first defined by Maxwell. It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit, *i.e.,*

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$= -P\frac{V}{\Delta V}$$

K is the property for solids, liquids and gases.

For gases, bulk modulus is of two types :

(i) Isothermal Bulk modulus of elasticity (K_i) which is equal to the pressure of gas (P).

(ii) Adiabatic Bulk modulus of elasticity (K_a) which is equal to γ P, where $\gamma = \dfrac{C_p}{C_v}$.

K is maximum for solids, less for liquids and least for gases.

- **Compressibility** is defined as the reciprocal of bulk modulus of elasticity. The pressure necessary to stop volume expansion of a piece of metal is $P = K\gamma\Delta T$, where K is bulk modulus of elasticity of metal, γ is the coefficient of volume expansion ($\gamma = 3\alpha$).
- **Modulus of Rigidity** (η) is defined as the ratio of tangential stress to the shearing strain, within the elastic limit, *i.e.,*

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

η is the characteristic of solid material only as the liquids and gases do not have fixed shape. η for liquid is zero.

- **Poisson's ratio** (σ). It is defined as the ratio of lateral strain to the longitudinal strain, *i.e.,*

$$\sigma = \frac{\text{Lateral strain } (\beta)}{\text{Longitudinal strain } (\alpha)}$$

$$= \frac{-\Delta D / D}{\Delta l / l} = -\frac{-\Delta D.l}{D.\Delta l}$$

Numerically value of σ lies between –1 and + $\frac{1}{2}$ but practical value of σ lies between 0 and + $\frac{1}{2}$.

For rubber, $\sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$, where L and V are the initial length and volume of the rubber tube respectively, dV = change in volume, dL = change in length.

Know the Terms

- **Ductile materials** are those materials which show large plastic range beyond the elastic limit, and which can be drawn into sheets and springs. Examples are copper, silver, iron, aluminium etc.
- **Brittle materials** are those materials which show very small range beyond the elastic limit. For such materials the breaking point lies close to the elastic limit. Examples are cast iron, glass etc.
- **Elastomers** are those materials for which stress and strain variation is not in straight line, within the elastic limit, and strain produced is much larger than the stress applied. Such materials have no plastic range. The breaking point lies very close to elastic limit. Example is rubber.

  The material which can be greatly stretched are called elastomers.
- **Elastic after effect :** The time delay in regaining the original configuration by the elastic body after the removal of a deforming force is called elastic after effect. It is least for quartz or phosphor bronze and maximum for glass. Elastic after effect is a temporary absence of elastic properties. A temporary loss of elastic properties due to continuous use of a body for long time is called **elastic fatigue.**

Know the Formulae

- Young's modulus $\qquad Y = \frac{Fl}{\pi r^2 \Delta l}$
- Bulk modulus $\qquad K = -\frac{FV}{A\Delta V} = -P\frac{V}{\Delta V}$
- Modulus of Rigidity $\qquad \sigma = \frac{FL}{A\Delta L}$
- Poisson's Ratio $\qquad \sigma = \frac{-\Delta D.l}{D.\Delta l}$

  For rubber $\qquad \sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$
- **Relation Among Various Elastic Constants :**

(i) Relation between σ, α and β,

$$\sigma = \frac{\beta}{\alpha}$$

where $\qquad \beta = dD/D$

and $\qquad \alpha = dl/l$

(ii) Relation between Y and α, $\qquad Y = \frac{1}{\alpha}$ when stress = 1 unit

(iii) Relation between η, α and β, $\qquad \eta = \frac{1}{2(\alpha + \beta)}$

(iv) Relation between Y, K and σ, $\qquad Y = 3K(1 - 2\sigma)$

(v) Relation between Y, η and σ, $\qquad Y = 2\eta(1 + \sigma)$

(vi) Relation between K, η and σ, $\sigma = \frac{3K - 2\eta}{2\eta + 6K}$

(vii) Relation between Y, K and η, $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$

- Elastic potential energy in stretched wire, $U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of wire}$
- Elastic potential energy per unit volume of wire,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} Y \times (\text{Strain})^2$$

- Interatomic force constant, (K) $K = r_0 \times Y$

Here, r_0 = interatomic distance

- Energy stored per unit volume of a strained body, $u = \frac{1}{2} Y \times (\text{Strain})^2$
- Work done in a streching wire $W = \frac{1}{2} \times \text{Load} \times \text{Extension.}$

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CHAPTER-10
MECHANICAL PROPERTIES OF FLUIDS

TOPIC-1
Fluids at Rest

Revision Notes

- **Pressure :**

(i) Pressure is defined as the thrust acting per unit area of the surface in contact with liquid, *i.e.*,

$$P = \frac{\text{Thrust(F)}}{\text{Area(A)}} = \frac{F}{A} = h\rho g$$

(ii) Liquid pressure is independent of shape of the liquid surface as well as area of the liquid surface, but depends upon height of liquid column.

(iii) Total pressure at a depth h below liquid surface is $P = h\rho g + P_0$, where P_0 is the atmospheric pressure.

(iv) S.I. unit of pressure is Nm^{-2} or pascal (denoted by Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

(v) Pressure is a scalar quantity because a liquid at rest exerts equal pressure in all directions at all points in the same horizontal plane.

- **Pascal's Law :** It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same. Pascal's law also states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of liquid provided the gravity effect is neglected.
- **Atmospheric pressure :**

(i) It is defined as the pressure exerted by atmosphere.

(ii) At S.T.P., the value of atmospheric pressure is 1.01×10^5 Nm^{-2} or 1.01×10^6 dyne/cm^2.

- **Archimedes' principle :** It states that when a body is immersed partly or wholly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Observed weight of body = True weight – Weight of liquid displaced.

If w is the observed weight of body of density ρ when immersed in a liquid of density σ, then

$$w = Mg - mg$$

$$= Ah\rho g - Ah\sigma g$$
$$= Ahg(\rho - \sigma)$$
$$= Ahg\rho\left(1-\frac{\sigma}{\rho}\right) = W\left(1-\frac{\sigma}{\rho}\right)_5$$

$\therefore$ True weight, $$W = \frac{\text{apparent weight}}{(1-\sigma/\rho)}$$

- **Laws of floatation :** It states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.
  - **(a)** When true weight of the body $W > w$ (weight of the liquid displaced), the body will sink to the bottom of the liquid. It will be so when the density of solid body (ρ) is greater than the density of liquid (σ), *i.e.* $\rho > \sigma$.
  - **(b)** When $W < w$, the body will rise above the surface of liquid to such an extent that the weight of the liquid displaced by immersed part of the body (*i.e.* upward thrust) becomes greater than the weight of the body. The body then will float. In this case the density of solid body is less than the density of liquid, *i.e.*, $\rho < \sigma$.
  - **(c)** When $W = w$, the body is at rest anywhere in the liquid. The body will float with its whole volume just immersed in the liquid. In this case the density of body is equal to density of liquid, *i.e.*, $\rho = \sigma$.

  There will be equilibrium of floating body when
  - **(i)** Weight of liquid displaced by the immersed part of body is equal to the weight of the body.
  - **(ii)** The centre of gravity of the body and the centre of buoyancy lie along the same vertical line.
  - **(iii)** If the centre of gravity of the body lies vertically below the meta centre then body is in stable equilibrium.

  The body will be in unstable equilibrium if centre of gravity lies vertically above the meta centre.

Know the Terms

- **Fluid** is the name given to a substance which begins to flow when external force is applied on it. It includes liquid and gas.
- **Thrust :** The total normal force exerted by liquid at rest on a given surface in contact with it is known as thrust of liquid on that surface. It is due to collision of molecules of liquid while moving at random, with the walls of the container and rebounding from them.
- **Density** of a substance is defined as the mass per unit volume of substance.
- **Relative density** is defined as the ratio of its density of the substance to the density of water at 4°C.
- **Buoyancy** is the upward force acting on the body immersed in a fluid.
- **Inter-molecular forces** is the forces between the molecules of substances.
- **Adhesive force :** is the force of attraction acting between molecules of two different materials. For example, the force acting between the molecules of water and glass.
- **Cohesive force :** is the force of attraction acting between molecules of the same material. For example, the force acting between the molecules of water or mercury etc.
- **Metacentre :** is a point where the vertical line passing through the centre of Buoyancy intersects the central line.
- **Inter-molecular Binding Energy** of a liquid is the minimum energy required to separate two molecules of a liquid from each others influence.

Know the Formulae

- Pressure $= \frac{F}{A} = h\rho g$ (due to h height of liquid)

  h = height, ρ = Density of liquid, g = Acceleration due to gravity.
- Gauge pressure = Total pressure – Atmospheric pressure
- For Hydraulic lift; $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

  F_1, F_2 = Forces on pistons of area of cross – sections A_1, A_2
- Density $= \frac{\text{Mass}}{\text{Volume}}$, Relative density $= \frac{\text{Density of substance}}{\text{Density of water at 4°C}}$
- **Archimedes' Principle :**

  Loss of weight of body in liquid = Weight of liquid displaced = Volume × Density of liquid × g
- **Law of floatation :**

  A body will float if, weight of body = Weight of liquid displaced.

TOPIC-2
Surface Energy & Surface Tension

Revision Notes

- **Surface Tension :** It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.
  - **(a)** Quantitatively, surface tension of a liquid is measured as the force acting per unit length of a line imagined to be drawn tangentially any where on the free surface of the liquid at rest. It acts at right angles to this line on both the sides and along the tangent to the liquid surface, *i.e.*, $S = F/l$.
  - **(b)** Surface tension of a liquid is also defined as the amount of work done in increasing the free surface of liquid at rest by unity at constant temperature, *i.e.*, $S = W/A$.
    or $W = S \times A$ = Surface tension × Area of liquid surface formed.
  - **(c)** Surface tension is a molecular phenomenon and it arises due to electromagnetic forces. The explanation of surface tension was first given by Laplace.
  - **(d)** S.I. units of surface tension is Nm^{-1} or Jm^{-2} and C.G.S. unit is dyne cm^{-1} or erg-cm^{-2}.
  - **(e)** Dimensional formula of surface tension = $[ML^0T^{-2}]$
  - **(f)** Surface tension is a scalar quantity as it has no specific direction for a given liquid.
  - **(g)** Surface tension does not depend upon the area of the free surface of liquid at rest.
- **Surface Energy :** It is defined as the amount of work done against the force of surface tension in forming the liquid surface of a given area at a constant temperature, *i.e.*,

  Surface energy = Work done = S.T. × Surface area of liquid.

  The S.I. unit of surface energy is Joule and C.G.S. unit is erg.
  - **(a)** If small drops combine together to form a big drop, the surface area decreases, so surface energy decreases. Therefore the energy is released. If this energy is taken by drop, the temperature of drop increases.
  - **(b)** If a big drop is splitted into number of smaller drops, the surface area of drops increases. Hence, surface energy increases. So energy is spent. If this energy is pronated by drop, the temperature of drop decreases *e.g.*, spray.

Know the Terms

- **Molecular range** is the maximum distance upto which a molecule can exert some measurable attraction on other molecules.
- **Sphere of influence** is an imaginary sphere drawn with a molecule as centre & molecular range as radius.
- **Surface film** is the top most layer of liquid at rest with thickness equal to the molecular range.
- **Angle of contact** between a liquid and solid in contact is defined as the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid.
- **Capillary tube** is a tube with a fine and uniform bore throughout its length.
- **Capillarity** is the phenomenon of rise or fall of liquid in a capillary tube.

Know the Formulae

- **Surface tension,** $S = F/l$
- **Surface energy,** E = Work done
- **Work done,** $W = S \times$ Increase in area
- **Excess of pressure inside the liquid drop is** $P = P_i - P_o = \frac{2S}{r}$; P_i = Pressure inside bubble
- **Excess of pressure inside the soap bubble is** $P = P_i - P_o = \frac{4S}{r}$; P_0 = Pressure outside bubble
- **Total pressure in the air bubble at a depth h below the surface of liquid of density ρ is**

$$P = P_o + h\rho g + \frac{2S}{r}$$

➢ **Ascent/Decent Formula :**

$$h = \frac{2S\cos\theta}{r\rho g}$$

where,

r = radius of capillary tube
ρ = density
S = Surface tension
θ = angle of contact

TOPIC-3
Viscosity & Bernoulli's Theorem

Revision Notes

➢ **Bernoulli's theorem :** Bernoulli's theorem state that the total energy per unit volume (pressure energy, P.E. and K.E.) per unit volume of an incompressible non-viscous liquid in steady flow remain constant throughout the flow of the liquid $P + \rho gh + \frac{1}{2}\rho v^2$ = constant.

➢ **Torricelli's theorem :** According to this theorem, velocity of efflux *i.e.*, the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

Know the Terms

➢ **Viscosity** is the property of liquid due to which a backward dragging force called viscous force act tangentially on the layer of the liquid in motion.
➢ **Terminal velocity** is the maximum constant velocity acquired by the body while falling freely in a viscous medium.
➢ **Streamline flow of a liquid** is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.
➢ **Streamline** is the actual path followed by the procession of particles in a steady flow which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point.
➢ **Tube of flow** is the bundle of streamlines having the same velocity of the liquid particle over any cross-section perpendicular to the direction of flow of liquid.
➢ **Laminar flow** is a flow in which the liquid moves in layers.
➢ **Turbulent flow** is a flow when a liquid moves with a velocity greater than its critical velocity, the motion of particles of liquid becomes disorderly or irregular.
➢ **Critical velocity** is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.
➢ **Reynold number** is a pure number which determines the nature of flow of liquid through a pipe.
➢ **Venturimeter** is a device used for measuring the speed of incompressible liquid and rate of flow of liquid through pipes.

Know the Formulae

➢ **Newton's viscous drag force :**

$$F = \pm \eta A \frac{dv}{dx}$$

η = coeff. of viscosity, A = area of layer of liquid, dv/dx = velocity gradient.

➢ **Poiseuille's Formula :**

$$V = \frac{\pi p r^4}{8\eta l}$$

p = Pressure difference across length l of horizontal tube of radius r & V = volume.

➢ **Stoke's Law :** $F = 6\,\pi\eta rv$

➢ **Terminal velocity :**

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where,

ρ = density of spherical body

σ = density of medium

r = radius of spherical body

η = coeff. of viscosity

- **Reynold's Number :**

$$R_N = \frac{\rho D v}{\eta}$$

Where, symbols have their usual meaning

- **Bernoulli's Theorem :**

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant} \quad \text{or} \quad \frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

$\frac{P}{\rho}$ = pressure energy per unit mass

$\frac{P}{\rho g}$ = Pressure head, h = gravitational head

gh = P. E. per unit mass

$\frac{v^2}{2g}$ = Velocity head

$\frac{1}{2}v^2$ = K.E. per unit mass

- **Venturimeter :** Volume of liquid flowing per second

$$V = a_1 a_2 \sqrt{\frac{2\rho_m gh}{\rho\left(a_1^2 - a_2^2\right)}}$$

a_1, a_2 = area of cross-section of bigger & smaller tube of venturimeter.
h = difference of pressure head at the two tube of venturimeter. ρ_m - density of liquid in manometer

- **Velocity of efflux :** $v = \sqrt{2gh}$

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CHAPTER-11

THERMAL PROPERTIES OF MATTER

TOPIC-1
Thermal Expansion & Heat Capacities

Revision Notes

- **Four Scales of Temperature :**

S. No.	Scale	Ice point	Steam point	No. of divisions	Smallest division
1.	Centigrade scale	0°C	100°C	100	1°C
2.	Fahrenheit scale	32°F	212°F	180	1°F
3.	Reaumer scale	0°R	80°R	80	1°R
4.	Thermodynamical scale of Absolute Kelvin scale	273 K	373 K	100	1 K

- **Thermal Expansion of solids. :** It is the phenomenon of expansion of solids on heating. It is of three types :
  (a) Linear Expansion : It is the increase in length of a solid on heating. α is called coefficient of linear expansion.
  (b) Area Expansion : It is the increase in surface area of a solid on heating. β is called coefficient of area expansion.
  (c) Volume Expansion : It is the increase in volume of a solid on heating. γ is called coefficient of volume expansion.
- **Expansion of liquids :**
  (a) Coefficient of real expansion of a liquid is defined as the real increase in volume of the liquid per unit original volume per °C rise in temperature. If γ_r is the coefficient of real expansion of a liquid, then

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

  (b) Coefficient of apparent expansion of a liquid is defined as the apparent increase in volume per unit original volume per °C rise in temperature. If γ_a is the coefficient of apparent expansion of a liquid, then

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Know the Terms

- **Heat** is a form of energy, which produces the sensation of warmth. The thermal energy in matter is present in the form of translational, rotational and vibrational energy of its atoms/molecules.
- **Temperature** of a body is a measure of degree of hotness/coldness of the body. This macroscopic property determines the direction of flow of heat, when the given body is placed in contact with some other body.
- Anomalous expansion of water is the volume of water decreases with increase in temperature from 0°C to 4°C.
- **Specific heat of a substance** is the amount of heat required to raise the temperature of unit mass of substance through unit degree.
- **Molar specific heat of a substance** is the amount of heat required to raise the temperature of 1 gm mole of substance through 1°C.
- **Heat capacity or thermal capacity** of a body is the amount of heat required to raise the temperature of whole body through 1°C or 1 K.
- **Water equivalent** is the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature & represented by W.
- **Change of state** is the conversion of one of the states of matter to another.
- **Latent heat** of a substance is the amount of heat required to change the state of unit mass of the substance at constant temperature (Q = ML). Its units are cal/g or joule/kg and its dimensions are $[L^2T^{-2}]$.

Know the Formulae

- **Temperature**
  (a) Relation between °C, °F and °R is

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$$

  (b) T K = (t°C + 273) or t°C = (T K – 273)
  (c) Temperature. diff. of 1°C = Temp. diff. of 1 K.
  (d) Normal temp. of a person is 98.6°F or 37°C.
  (e) The temp. of – 40° is same in °C and °F.
- **Thermal expansion**
  (a) Coeff. of linear expansion $\alpha = \frac{\Delta L}{L\Delta T}$
  (b) Coeff. of Area expansion $\beta = \frac{\Delta S}{S(\Delta T)}$
  (c) Coeff. of volume expansion $\gamma = \frac{\Delta V}{V(\Delta T)}$
  (d) $\beta = 2\alpha$; $\gamma = 3\alpha$
  (e) In liquids. $\gamma_r = \gamma_a + \gamma_g$
  where,
  γ_r = Coefficient of real expansion of liquid

γ_a = Coefficient of apparent expansion of liquid
γ_g = Coefficient of expansion of vessel

- **Specific heat.** $\Delta Q = ms\Delta T$
- **Molar specific heat** $C = M \times s$
- **Latent heat** $\Delta Q = ML$
  L = Latent heat
- **Specific heat of gases** $C_p - C_V = R$, Here $R = \frac{PV}{T}$

TOPIC-2
Heat Transfer

Revision Notes

- **Thermal Conductivity :**

  (i) Coefficient of Thermal Conductivity : It is equal to rate of flow of heat per unit area per unit temperature gradient across the solid at steady state. It is represented by K & its value depends on nature of material of solid.

$$K = \frac{\Delta Q \Delta x}{\Delta T A}$$

$$K = \Delta Q, \text{ when } \left(\frac{\Delta x}{\Delta T}\right) = 1, A = 1$$

  (ii)Thermal resistance corresponds to electrical resistance (V/i) and is given by Temperature diff/rate of flow of heat *i.e.*,

$$R_{Th} = \frac{T_1 - T_2}{dQ/dt} = \frac{x}{KA}$$

  where symbols have their usual meaning.
  Clearly, greater is the value of K, smaller is the thermal resistance.

- **Total emittance or emissive power** of a body at a certain temperature is the total amount of thermal energy emitted per unit time per unit area of the body for all possible wavelengths. It is represented by e'

$$e' = \int_0^\infty e_\lambda d\lambda$$

- **Emissivity** (ε) of a body at a given temperature is the ratio of emissive power of the body (e) to the emissive power of perfectly black body (E) at that temperature,

  i.e., $\varepsilon = \frac{e}{E}$

  Similarly, we can define monochromatic absorptance or spectral absorptive power. Total absorptance or absorbing power

$$a = \int_0^\infty a_\lambda d\lambda$$

- **Kirchhoff's Law.** From this law, at a given temperature and for a given wavelength, the ratio of spectral emissive power (e_λ) to spectral absorptive power (a_λ) for all bodies is constant which is equal to spectral emissive power of a perfectly black body (E_λ) at the same temperature and for the same wavelength, *i.e.,* $\frac{e_\lambda}{a_\lambda} = E_\lambda$ clearly, $e_\lambda \propto a_\lambda$ it means good emitters are good absorbers. The law implies that at a particular temperature, a body can absorb only those wavelengths, which it is capable of emitting.

- **Wien's law :** From this law, the wavelength (λ_m) corresponding to which energy emitted/sec/area by a perfectly black body is maximum, is inversely proportional to the absolute temperature (T) of the black body.

$$\lambda_m \propto \frac{1}{T}$$

or $$\lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is known as Wien's constant $b = 2.898 \times 10^{-3}$ mK.

- **Newton's law of cooling.** According to this law, when difference in temperature of a liquid and its surroundings is small (~ 30°C), then the rate of loss of heat of the liquid is directly proportional to difference in temperatures of the liquid and the surroundings, *i.e.*,

$$-\frac{dQ}{dt} \propto (\theta - \theta_0)$$

or $$-\frac{dQ}{dt} = K(\theta - \theta_0)$$

where K is constant of proportionality.

- **Stefan's law :** From this law, the total energy emitted/sec/area (E) by a perfectly black body corresponding to all wavelengths is directly proportional to fourth power of the absolute temp. (T) of the body *i.e.*

$$E \propto T^4$$

or $$E = \sigma T^4$$

There σ is a constant of proportionality and is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-8} \text{ watt m}^{-2}\text{K}^{-4}$$

If Q is the total amount of heat energy emitted by the black body, then by definition,

$$E = \frac{Q}{At}$$

$$\therefore \quad Q = At \times E = At(\sigma T^4)$$

If the body is not perfectly black and has an emissivity e, then $Q = eAt(\sigma T^4)$

- **Stefan Boltzman law :** From this law, the net amount of radiation emitted per second per unit area of a perfectly black body at temperature T is equal to difference in the amounts of radiation emitted/sec/area by the body and by the black body enclosure at T_0.

$$\Rightarrow \quad E' = E - E_0$$

As $$E = \sigma T^4$$

and $$E_0 = \sigma T_0^4$$

$$\therefore \quad E' = \sigma T^4 - \sigma T_0^4$$
$$= \sigma(T^4 - T_0^4)$$

Proceeding as above, total energy lost

$$Q' = E'At$$
$$= At\sigma(T^4 - T_0^4)$$

When the body and enclosure are not perfectly black and have emissivity ε, then

$$Q' = \varepsilon At\sigma(T^4 - T_0^4)$$

Know the Terms

- **Conduction** is the mode of transfer of heat from one part of the body to another, from particle to particle in the direction of fall of temperature without any actual movement of heated particles.
- **Thermal convection** is the phenomenon of transfer of heat by actual mass motion of the medium. All liquids and gases are heated by convection.
- **Radiation** is the phenomenon of transfer of heat from source to the receiver without any actual movement of source or receiver and without heating the intervening medium. For example, heat comes to us from the sun through radiation.
- **Energy flux** is the energy flowing per second per unit area normal to surface. Its unit is watt/m^2.
- **Energy density** is the total energy per unit volume. Its unit is joule/m^3.
- **Steady-State** is the state of rod in which temperature of each part becomes constant & there is no further absorption of heat.
- **Variable state** is the state of rod in which temperature of every cross-section of the rod goes on increasing.
- **Monochromatic emittance or spectral emissive power** of a body corresponding to a particular wavelength λ at

a particular temperature is the amount of radiant energy emitted per unit time per unit surface area of the body within unit wavelength interval around λ. It is represented by e_λ.

- **Perfectly black body** is that which absorbs all the radiations incident upon it. Thus absorptive power of a perfectly black body is unity (*i.e.*, 100%). When such a body is heated to high temperature, it would emit radiations of all wavelengths.

Know the Formulae

- **Rate of conduction of Heat,** $\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$

where,

$$\frac{\Delta T}{\Delta x} = \text{temperature gradient}$$

- **Thermal resistance,** $R_{Th} = \frac{T_1 - T_2}{dQ/dt}$
- **Emissive power,** $e' = \int_0^\infty e_\lambda d\lambda$
- **Emissivity,** $\varepsilon = \frac{e}{E}$

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UNIT-VIII
THERMODYNAMICS

CHAPTER-12
THERMODYNAMICS

TOPIC-1
Heat, Zeroth and First Law of Thermodynamics

Revision Notes

- **Facts About Specific Heat :** The amounts of heat required to raise the temperature of unit mass of a substance by 1°C.

(a) Specific heat at constant volume (C_V) = amount of heat required to raise the temperature of one gram of gas through 1°C at constant volume.

(b) Specific heat at constant pressure (C_P) = amount of heat required to raise the temperature of one gram of gas through 1°C at constant pressure.

(c) Molar specific heat at constant volume/pressure = amount of heat required to raise the temperature of one gram mole of the gas through 1°C at constant volume/pressure. It is represented by c_v and c_p respectively. Thus

$$C_V = M \times c_v$$

and

$$C_p = M \times c_p$$

where M is molecular weight of the gas. c_v and c_p are measured in cal/gram mole/°C.

(d) Out of the two principal specific heats of a gas,

$$C_p > C_v$$

(e) The ratio of two principle specific heats of a gas is always greater than 1, *i.e.*,

$$\gamma = \frac{C_p}{C_v} > 1$$

(f) The value of γ depends upon nature of the gas,

$$\gamma = 1 + \frac{2}{n}$$

where n is the number of degrees of freedom of the molecules of the gas,

$$n = 3A - R$$

where A is no. of atoms in each molecule and R is no. of independent relations among the atoms in a molecule.

➢ **Internal energy :** The energy possessed by the molecules of a gas by virtue of their particular configuration and molecular motion is called internal energy of the gas. It is of two types :

(a) Internal potential energy (U_P) : It is due to molecular configuration, *i.e.,* due to mutual interaction of atoms/ molecules.

(b) Internal kinetic energy (U_k) : It is due to motion of the molecules of the gas. Hence,

$$U = U_P + U_K$$

Internal energy of a **real gas** depends on volume of the gas and also on temperature of the gas.

➢ **Four thermodynamical operations :** There are following four operations :

(a) Isothermal changes, where the temperature remains constant. The pressure and volume of a given mass of gas changes.

Two essential conditions are :

(i) Walls of container must be perfectly conducting.

(ii) Changes must be slow.

(b) Adiabatic changes, where the heat content of a gaseous system remains constant. The pressure and volume of given mass of gas change with consequent change in temperature.

Two essential conditions are :

(i) Walls of container must be perfectly insulating.

(ii) Changes must be sudden.

(c) Isobaric changes, where pressure is kept constant.

(d) Isochoric changes, where volume is kept constant.

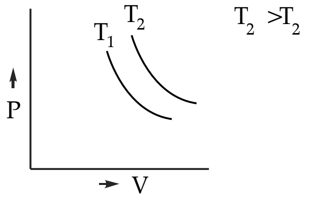
➢ **Characteristics of isothermal process :**

(a) T = constant or $\Delta T = 0$

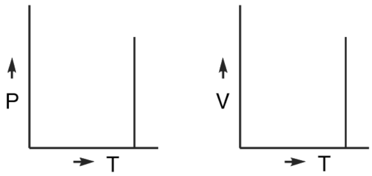
(b) U = constant or $\Delta U = 0$

(c) Equation of isothermal changes is PV = constant

(d) Variation of P with V at constant temperature is represented by Isothermal curves, which are rectangular hyperbola as shown in following figures.



(e) The P-T and V-T graphs are straight lines perpendicular to temperature axis. They are shown in figures.



(f) Bulk modulus of elasticity under isothermal conditions is given by

$$K_i = \frac{\Delta P}{\Delta V/V} = -P$$

➢ **Characteristics of adiabatic process :**

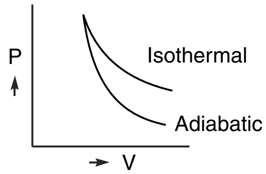
(a) Q = constant or $\Delta Q = 0$

(b) If S represents entropy, then from $\Delta S = \dfrac{\Delta Q}{\Delta T} = 0$

i.e., there is no change of entropy in an adiabatic change. That is why an adiabatic process is called iso-entropic process.

(c) Equation of adiabatic changes is PV^γ = constant, where $\gamma = \dfrac{C_P}{C_V}$

(d) The variation of P with V at constant heat content is represented by an Adiabatic curve, shown in figure, which is also a rectangular hyperbola.



(e) Since, it is clear from figure an adiabatic curve is steeper than an isothermal curve. In fact,

slope of adiabatic curve = γ-times the slope of isothermal curve.

(f) Bulk modulus of elasticity, under adiabatic conditions is given as

$$K_a = \frac{\Delta P}{\Delta V/V} = -\gamma P = \gamma K_i$$

- **Zeroth law of thermodynamics** corresponds to the concept of temperature, thermal equilibrium of a system. According to this law, if two systems A and B are separately in thermal equilibrium with a third system C, then A and B will also be in thermal equilibrium with each other.
- **First law of thermodynamics** is basically the law of conservation of energy. According to this law, when a certain amount of heat energy (dQ) disappears, an equivalent amount of energy appears in some other form. When (dU) is small increase in internal energy and (dW) is small amount of external work done by the system in expansion, then

$$dQ = dU + dW$$

Know the Terms

- **Thermodynamics** is a branch of physics which deals with conversion of heat energy into mechanical work and vice-versa.
- **Thermodynamical system & thermodynamical parameters.**
  A gaseous system is called a thermodynamical system. The state of the system is represented in terms of pressure (P), volume (V), temperature (T) and heat content (Q) of the gas. These four quantities are called thermodynamical parameters of the system.
- **Thermodynamic equilibrium :** A system is said to be in thermodynamic equilibrium when macroscopic variables like pressure, volume, temperature, mass, composition etc. that characterise the system do not change with time.
- **Heat** is the transfer of kinetic energy from one medium or object to another, or from an energy source to a medium or object.
- **Temperature** is the degree of hotness or coldness of a body.
- **Open system :** Exchanges both energy & matter with surroundings.
- **Closed system :** Exchanges only energy with surroundings.
- **Isolated system :** Exchanges neither energy nor matter with surroundings.
- **Equation of state** is the equation connecting pressure, volume and temperature of the gas.

Know the Formulae

- **Mayer's relation :** $C_p - C_v = \frac{R}{J}$

where R is gas constant for one gram mole of the gas and J = Mechanical equivalent of heat.

- **Equation of state for :**

(a) an ideal gas : $PV = RT$
(b) a real gas : $(P + a/V^2)(V - b) = RT$ } for 1 mole
(c) an isothermal process : $PV = \text{Constant.}$
(d) an isobaric process : $\frac{V}{T} = \text{Constant.}$
(e) an isochoric process : $\frac{P}{T} = \text{Constant.}$
(f) an adiabatic process :

$$PV^\gamma = \text{Constant;}$$
$$TV^{\gamma-1} = \text{Constant and}$$
$$\frac{P^{\gamma-1}}{T^\gamma} = \text{constant.}$$

- **Work done during expansion of gas :** $dW = PdV$ (for constant pressure)

or $W = \int_{V_1}^{V_2} PdV$ (for variable pressure)

(a) In an isothermal process : $W = 2.3026\ RT \log_{10}\left(\frac{V_2}{V_1}\right)$

$$W = 2.3026\ RT \log_{10}\left(\frac{P_1}{P_2}\right)$$

(b) In an adiabatic process : $W = \frac{R}{1-\gamma}(T_2 - T_1)$

$$= \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

Other expressions for work done by a gas in adiabatic expansion may by expressed as

$$W = \frac{R}{1-\gamma}(T_2 - T_1)$$

$$= \frac{RT_1}{\gamma-1}\left(1 - \frac{T_2}{T_1}\right)$$

$$W = \frac{(C_P - C_V)}{\left(\frac{C_P}{C_V} - 1\right)}(T_1 - T_2) \qquad \left[\because R = C_P - C_V \text{ and } \gamma = \frac{C_P}{C_V}\right]$$

$$= C_V(T_1 - T_2)$$

$$= \frac{C_P}{\gamma}(T_1 - T_2)$$

(c) In an isobaric process : $W = P(V_2 - V_1)$

(d) In a isochoric process :
As V = constant, $dV = 0, \therefore W = 0$

(e) In a non-cyclic process :
W = area of P-V curve = area enclosed between the PV curve and volume axis.
The value of W would depend upon the **path followed** by the system in going from initial state to final state.

(f) In a cyclic process :
W = area enclosed by the closed loop representing the cyclic process.
W is positive when loop is traced in clockwise direction and W is negative, when loop is traced in anti clockwise direction.
When a gas **expands,** work is done by the gas. It is taken as **Positive.**
When a gas is **compressed,** work is done on the gas. It is taken as **Negative.**

- If heat is converted into work or work is converted into heat, then W = JQ, where J = Joule's mechanical equivalent of heat = 4.2 joule/calorie.

  The value of W could be mgh or $\frac{1}{2}mv^2$.

TOPIC-2
Second Law of Thermodynamics

Revision Notes

- **Cyclic & Non-Cyclic processes :**
  (i) In a cyclic process, the system returns to its initial state after passing through various stages of pressure, volume and temperature. P-V curve representing a cyclic process is a closed curve.
  (ii) In non-cyclic process, the series of changes involved do not return the system back to its initial state.
- **Heat Engine :** It is a device which converts heat energy into mechanical energy. It consists of 3 parts :
  (a) Source of heat at higher temperature.
  (b) Working substance.
  (c) Sink of heat at lower temperature.

Types of Heat Engine :

1. External combustion engine : Heat is produced by burning fuel outside the main body.

2. Internal combustion engine : Heat produced by burning fuel inside the main body.

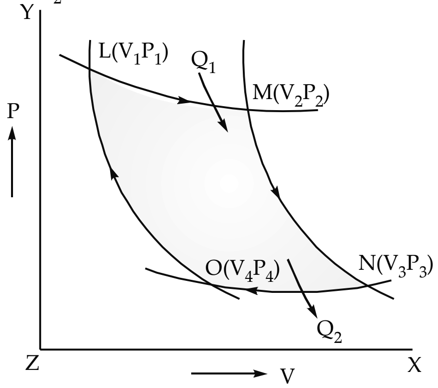
- **Second Law of Thermodynamics** represents the direction of flow of heat. According to **Clausius statement**, it is impossible for a self-acting machine unaided by any external agency, to transfer heat from a body at lower temperature to a body at higher temperature.

  So, heat can be transferred from lower temperature to higher temperature only when some external work is done or energy is supplied to the system. For example, a refrigerator cools things only when electric energy is supplied to it. Further, we deduce from second law, that efficiency of any heat engine can never be 100%.

  According to **Kelvin-Plank statement**, No process is possible whose sole result in the absorption of heat from a reservoir and the complete conversion of the heat into work.

- **A Carnot engine** is an ideal heat engine, which consists of a **source** of infinite thermal capacity which is maintained at a constant high temperature T_1 and a **sink** of infinite thermal capacity maintained at a constant low temperature T_2 and an ideal gas acting as the working substance. It consists of a cylinder fitted with an insulating frictionless piston. The cylinder has a conducting base and insulating walls. It can be placed on an insulating pad.

  Carnot engine : Works on the principle of Carnot cycle made up of four stages : Isothermal expansion LM; Adiabatic expansion MN; Isothermal compression NO; Adiabatic compression OL as represented in given figure. If Q_1 is the amount of heat absorbed per cycle from the source at temperature T_1 ; Q_2 is the amount of heat rejected per cycle to the sink at temperature T_2;



- **Carnot's Theorem :**

  (a) Working between two given temperatures. T_1 of hot reservoir and T_2 of cold reservoir, no engine can have efficiency more than that of carnot engine.

  (b) The efficiency of carnot engine is independent of the nature of working substance.

Know the Terms

1. **Reversible process** is a process which can be reversed back to initial state.
2. **Irreversible process** is a process which cannot be traced back in opposite direction.
3. **Entropy** of a system is a measure of its molecular disorder. The greater the disorder, greater is the entropy.
4. **Perpetual motion machine of first kind** disobeys first law of thermodynamics and **perpetual motion machine of second kind** disobeys second law of thermodynamics.

Know the Formulae

1. **Work done :**

   in cyclic process $dQ = dW$

   in non- cyclic process $dQ \neq dW$

2. **Change in Entropy** $\Delta S = \dfrac{\Delta Q}{T} = \dfrac{\text{Heat absorbed}}{\text{Absolute temperature}}$

3. **Efficiency of Heat Engine**

   i.e., $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

   where, W = useful work done/cycle by the engine

   Q_1 = amount of heat energy absorbed/cycle from the source

   Q_2 = amount of heat rejected/cycle to the sink

   and $W = Q_1 - Q_2$

   Where T_1 is temperature of source; T_2 is temperature of sink, then

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$\therefore$ $$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

4. Efficiency of Carnot's Engine $$W = Q_1 - Q_2$$

Efficiency, $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

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UNIT-IX
BEHAVIOUR OF PERFECT GASES AND KINETIC THEORY OF GASES

CHAPTER-13
KINETIC THEORY

TOPIC-1
Equation of State & Kinetic Theory of Gases

Revision Notes

- **Ideal Gases :**
  - **(a)** Ideal gas or perfect gas is that gas which strictly obeys gas laws, like Boyle's law, Charles's law etc.
  - **(b)** For ideal gas, the size of the gas molecules is almost zero and the volume of the gas molecule is also almost zero.
  - **(c)** There is no force of attraction or repulsion amongst the molecules of an ideal gas.
  - **(d)** There is no intermolecular potential energy for the molecules of an ideal gas.
  - **(e)** The molecules of an ideal gas posses only kinetic energy.
  - **(f)** The ideal gas cannot be liquefied or solidify, which supports the absence of intermolecular forces of ideal gas at very low pressure and high temperature.
  - **(g)** The internal energy of an ideal gas does not depend on volume.
  - **(h)** The internal energy of an ideal gas depends upon the temperature alone.
  - **(i)** The specific heat of an ideal gas is independent of temperature.
  - **(j)** No gas available in the universe is strictly an ideal gas.
  - **(k)** The gases such as H_2, O_2, N_2 etc. and monoatomic inert gases behave very similar to ideal gases at very low pressure and high temperature
  - **(l)** The real gases at low pressure and high temperature behave as ideal gases due to negligible intermolecular force of attraction and volume of gas molecules.
- **Real Gases :**
  - **(a)** Real gases which are actually found in nature are known as real gases.
  - **(b)** The size of the molecules of a real gas is finite and hence the volume of the molecules of a real gas is finite.
  - **(c)** There is a force of attraction or repulsion between the molecules of a real gas. The intermolecular force between molecules is attractive for large intermolecular separation and repulsive for small intermolecular separation.
  - **(d)** The molecules of a real gas have potential energy as well as kinetic energy.
  - **(e)** The internal energy of a real gas depends on pressure, volume and temperature of the gas.
  - **(f)** The real gas can be liquified and solidified.
  - **(g)** The real gas do not obey gas equation but obey Van der Waal's gas equation :

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where a and b are Van der Waal's gas constants of a real gas and n is the number of moles.

(h) The real gases like CO_2, NH_3, SO_2 etc. obey Van der Waal's equation at high pressure and low temperature.

(i) In Van der Waal's equation, the value of a depends upon the intermolecular force and the nature of the gas.

(j) The value of b depends upon the size of the gas molecules and represents the volume occupied by the molecules of a gas.

(k) In Van der Waal's equation $(V - b)$ shows volume available to the molecules of the real gas, which is the effective volume of the gas.

(l) Real gases do not obey the gas laws at all temperatures.

- **Boyle's law :** It states that the volume V of the given mass of a gas is inversely proportional to its pressure P, when temperature is kept constant, *i.e.*,

$$V \propto 1/P \text{ or } V = K/P \text{ ...(Here, T is constant)}$$

or $$\mathbf{PV = K = Constant.}$$

- **Charles's law :** It states that the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its kelvin temperature, *i.e.*, $V \propto T$, if P is constant.

or, $$V = KT \text{ ...(Here, P is constant)}$$

or, $$\frac{V}{T} = \mathbf{K = Constant.}$$

- **Assumptions of Kinetic Theory of Gases :**

(a) A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for different gases.

(b) The molecules of a gas are in a state of continuous, rapid and random motion.

(c) The volume occupied by the molecules is negligible in comparison to the volume of the gas.

(d) The molecules do not exert any force of attraction or repulsion on each other, except during collision.

(e) The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic.

(f) Molecular density is uniform throughout the gas.

(g) A molecule moves along a straight line between two successive collisions.

(h) The collisions are almost instantaneous.

Know the Terms

- **Gram/mole and kilogram/mole :**
  - **(i)** The molecular weight expressed in gram is known as gram/mole (g/mol). The molecular weight expressed in kilograms is known as kilogram/mole (kg/mol).
  - **(ii)** The mass of 1 mole of a gas equal to its molecular weight in gram. And 1 kg/mol = 1,000 g/mole.
- **Most probable speed** of the molecules of a gas is that speed which is possessed by maximum fraction of total number of molecules of the gas.
- **Mean speed or average speed** is the average speed with which molecules of a gas move.
- **Root mean square speed** is defined as the square root of the mean of the squares of random velocities of individual molecules of a gas.

Know the Formulae

- **Boyle's Law :** $PV = \text{constant}$
- **Charles's Law :** $\frac{V}{T} = \text{constant}$
- **Standard gas equation :** $PV = nRT$

  where n is the number of moles contained in the given ideal gas of volume V, pressure P and temperature T.
- **Gas constant :**

  (i) R is a universal gas constant and r is a gas constant for 1 gram of a gas.

  (ii) The universal gas constant is defined as the work done by (or on) a gas per mole per Kelvin *i.e.*

$$R = \frac{PV}{nT}$$

$$= \frac{\text{Pressure} \times \text{Volume}}{\text{No. of moles} \times \text{Temperature}}$$

$$= \frac{\text{Work done}}{\text{No. of moles} \times \text{Temperature}}$$

  (iii) The value of R for every gas at S.T.P. = $8{\cdot}31 \text{ J mole}^{-1} \text{ K}^{-1} = 1.98 \text{ cal. mol}^{-1} \text{ K}^{-1}$.

(iv) Dimensional formula for R = $[ML^2T^{-2}K^{-1}\,mol^{-1}]$.

- **Most probable speed :** $c_{mp} = \sqrt{\dfrac{2k_BT}{m}}$
- **Average speed** $c_{av} = \sqrt{\dfrac{8k_BT}{\pi m}}$
- **Root mean square speed** $c_{rms} = \sqrt{\dfrac{3k_BT}{m}}$

k_B = Boltzman constant, T = Temperature, m = mass

- **Ratio among speeds,** $c_{mp} : c_{av} : c_{rms} = \sqrt{2} : \sqrt{\dfrac{8}{\pi}} : \sqrt{3}.$
- **Van der Waal's equation for one mole of a gas,** $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$

TOPIC-2
Law of Equi-partition of Energy & Brownian Motion

Revision Notes

- **Gay Lussac's Law or Regnault's Law :** When volume of a certain mass of a gas is kept constant, the pressure P exerted by gas is directly proportional to temperature T of gas *i.e.* $P \propto T$.
- **Avogadro's Law :** It states that equal volumes of all gases under identical conditions of temperature and pressure contain the same no. of molecules. $n_1 = n_2$
- **Graham's Law of Diffusion—**It states that rates of diffusion of two gases are inversely proportional to the square roots of their densities $r \propto \dfrac{1}{\sqrt{\rho}}$ or $\dfrac{r_1}{r_2} = \sqrt{\dfrac{\rho_2}{\rho_1}}$
- **Dalton's Law of Partial Pressure :** It states that total pressure exerted by a mixture of non-reactive ideal gases is equal to sum of partial pressures which each would exert, if it alone occupied the same volume at the given temperature.

$$P_1 + P_2 + P_3 \,................ = P$$

- **Law of Gaseous Volumes :** It states that when gases react together, they do so in volumes which will be a simple ratio to one another and also to the volumes of product.
- **Law of Equipartition of energy :** It states that the energy for each degree of freedom in thermal equilibrium is $\dfrac{1}{2}k_BT$.
- **Brownian Motion :** It is defined as continuous zig-zag motion of particles of macroscopic size ($\approx 10^{-5}$ m) suspended in water or air or some other fluid. Brownian motion increases :

(a) When size of suspended object is smaller.
(b) When density of fluid is smaller.
(c) When temperature of medium is higher.
(d) When viscosity of medium is smaller.

Know the Terms

- **Pressure** exerted by gas is due to continuous bombardment of gas molecules against the walls of container.
- **Degrees of freedom** of a dynamic system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system.
- **Mean free path** is the average distance covered between two successive collisions by the gas molecule moving along the straight line.
- **Absolute zero of temperature** may be defined as that temperature at which the root mean square velocity of gas molecules reduces to zero.

Know the Formulae

- **Pressure exerted by Ideal Gas.** $P = \frac{1}{3}\frac{mnc^2}{V}$

  or $P = \frac{1}{3}\frac{M}{V}c^2 = \frac{1}{3}\rho c^2$

- **Relation between Pressure & K.E. of gas**

  $$PV = \frac{2}{3}E$$

- **Average K.E. of translation of 1 mole**

  $$= \frac{1}{2}Mc^2 = \frac{3}{2}RT.$$

- **Average K.E. of translation per molecule of gas**

  $$= \frac{1}{2}mc^2 = \frac{3}{2}k_BT.$$

- **Boyle's Law** PV = Constant at constant T.
- **Charles's Law** $V \propto c^2 \therefore V \propto T$ at constant pressure
- **Avogadro's Law** $n_1 = n_2$ at equal temperature , pressure and volume.
- **Graham's Law of diffusion** $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$
- **Dalton's Law of partial pressure** $P_1 + P_2 + P_3 + ... = P$
- **Degrees of freedom** For
  (a) Mono-atomic gas = 3
  (b) Di-atomic gas = 5
  (c) Tri-atomic gas = 7
  (d) Non-linear triatomic gas = 6
- **Law of equipartition of energy** $E_t = \frac{1}{2}k_BT$
- **Specific Heat Capacity of :**

  (a) Monoatomic gas $\gamma = \frac{C_P}{C_V}$ or $\gamma = \frac{5}{3} = 1.67$

  (b) Diatomic gas $\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$

  (c) Triatomic gas

  Linear gas molecules $\gamma = \frac{9}{7} = 1.28$

  Non-linear gas molecules $\gamma = \frac{4}{3} = 1.33.$

  (d) Polyatomic gas $\gamma = \left(1 + \frac{2}{n}\right)$, where n is the degree of freedom

  Specific heat for
  (i) Solids $c = 3R = 24.93$ J $Mole^{-1}$ K^{-1}
  (ii) Water $c = 9R = 74.7$ J $mole^{-1}$ K^{-1}

- **Mean free path**

  $$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n}{n} = \frac{ct}{n}$$

  $$\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{k_BT}{\sqrt{2}\,\pi d^2 P}$$

  Here, d is diameter, P is the pressure, T is temperature

UNIT-X
OSCILLATIONS AND WAVES

CHAPTER-14
OSCILLATIONS

TOPIC-1
Periodic Functions & Simple Harmonic Motion(S.H.M.)

Revision Notes

- **Harmonic Oscillations :** Those oscillations which can be expressed in terms of single harmonic function. *i.e.* (sine function or cosine function).

$$y = a\sin \omega t \text{ or } y = a\cos \omega t.$$

- **Non-Harmonic Oscillations :** Those oscillations which cannot be expressed in terms of single harmonic function *i.e.*,

$$y = a\sin \omega t + b\sin 2\omega t.$$

- **Periodic Functions :** Those functions which are used to represent periodic motion *i.e.*,

$$f(t) = f(t + T) = f(t + 2T).$$

sine & cosine functions are periodic functions.
- **Phase :** Phase of vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of particle at that instant with respect to its mean position.
- **Some Facts :**

(a) In oscillatory motion, the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalised equation of motion of the vibrating particle.
(b) When the displacement of the particle executing a vibratory motion is represented by $y = a\sin(\omega t + \phi)$, then $(\omega t + \phi)$ is called phase of the vibrating particle.
(c) ϕ is called the initial phase of the vibrating particle.

- **Simple Harmonic Motion :** It is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean (*i.e.*, equilibrium) position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean (*i.e.*, equilibrium) position at that instant, *i.e.*,

$$F = -ky$$

where k is known as force constant. Here negative sign represent that the restoring force (F) is always directed towards the mean position.
- **Geometrical interpretation of S.H.M. :** S.H.M. is defined as the projection of a uniform circular motion on any diameter of a circle of reference.

(a) S.H.M. may be linear and angular S.H.M.
(b) The linear S.H.M. is always along a straight line about a fixed point on a line, whereas the angular S.H.M. is always along an arc of a circle about a fixed point on the arc.
(c) The linear S.H.M. is controlled by force law, where $F = -ky$, where k is the restoring force constant, *i.e.*, force per unit displacement.
(d) The angular S.H.M. is controlled by torque law, where $\tau = -C\theta$, where C is the restoring torque constant, *i.e.*, restoring torque per unit twist.

- **Characteristics of S.H.M. :**

(a) **Displacement :** The displacement of a particle executing linear S.H.M. at an instant is defined as the distance of the particle from the mean position at that instant.
(b) **Velocity :** is defined as the time rate of change of the displacement of the particle at the given instant.
(c) **Amplitude :** The maximum displacement on either side of mean position.
(d) **Acceleration :** It is defined as the time rate of change of the velocity of the particle at the given instant.
(e) **Time Period :** It is defined as the time taken by the particle executing S.H.M. to complete one vibration.

- **Restoring Force & Force Constant :** Force constant is the force required to give unit displacement to the body.

$$F = -ky$$ Here, k is force constant.

- **Simple Pendulum :** It is most common example of S.H.M. An ideal simple pendulum consists of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

$$\text{Time period,} = 2\pi\sqrt{\frac{l}{g}}$$

Know the Terms

- **Periodic motion :** The motion which is identically repeated after a fixed interval of time. The time interval after which the motion is repeated is known as period of motion. *e.g.* The revolution of earth around the sun, its period is one year.
- **Oscillatory motion or vibratory motion :**

(a) The motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.

(b) In such a motion, the body is confined within well defined limits (called extreme positions) on either side of mean position. *e.g.*,
The motion of the pendulum of a wall clock is oscillatory motion.

- **Inertia Factor—**

Linear S.H.M : Inertia factor stands for mass of the body executing S.H.M.

Angular S.H.M : Inertia factor stands for moment of Inertia of the body executing S.H.M.

Some terms related to periodic motion :

Time period : It is the least interval of the time after which the periodic motion of a body repeats itself & is denoted by T.

Frequency : It is defined as the no. of periodic motions executed by the body per second.

Angular frequency : It is equal to the product of frequency of the body with factor 2π. *i.e.* $\omega = 2\pi\nu$.

Displacement : It is the change in physical quantity under consideration with time in a periodic motion.

- **Spring Factor—**

Linear S.H.M : Spring factor stands for force per unit displacement.

Angular SH.M : Spring factor stands for restoring torque per unit twist.

Know the Formulae

- **Periodic Motion.**

(a) Frequency, $\nu = \frac{1}{T}$

(b) Angular Frequency, $\omega = \nu \times 2\pi$

or $\omega = \frac{2\pi}{T}$

- $$\text{Phase} = (\omega t + \phi) = \left(\frac{2\pi t}{T} + \phi\right) = (2\pi\nu t + \phi)$$

or $\phi' = \frac{2\pi t}{T} + \phi$; Here, ϕ' = phase at time t and ϕ = initial phase

- **Simple Harmonic Motion :**

(a) Differential Equation,

(i) Linear S.H.M. $= \frac{d^2y}{dt^2} + \omega^2 y = 0$, where $\omega^2 = k/m$, here, m is the mass of the body

(ii) Angular S.H.M. $= \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$, where $\omega^2 = C/I$, here, I = moment of inertia

(b) General *eq*uation—

(i) Linear S.H.M. $y = y_0 \sin(\omega t + \phi)$

(ii) Angular S.H.M. $\theta = \theta_0 \sin(\omega t + \phi_0)$

(c) Displacement, $y = A\sin \omega t$

or $y = A\cos \omega t$

(d) Velocity, $v = \omega\sqrt{A^2 - y^2}$

(e) Acceleration, $a = \frac{dv}{dt} = -\omega^2 A\sin \omega t$

(f) Time Period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ or $2\pi\sqrt{\frac{I}{C}}$

➢ **Oscillations :**

(a) Loaded Spring :
Horizontal and vertical direction

$$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{k}}$$

Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(b) For simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

(c) Loaded Spring Combinations :

Case I : Two springs in parallel, $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$,

If, $k_1 = k_2 = k$, $T = 2\pi\sqrt{\frac{m}{2k}}$

Case II : Two springs in series, $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 \times k_2}}$,

If, $k_1 = k_2 = k$, $T = 2\pi\sqrt{\frac{2m}{k}}$

➢ **Spring Constant :** $k = \frac{F}{y}$

(i) In parallel, $k = k_1 + k_2$,

(ii) In series, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

TOPIC-2
Energy, Forced & Damped Oscillations

Revision Notes

➢ **Energy of S.H.M :** A particle executing S.H.M. possesses two types of energy :

(a) Potential Energy : This energy is on account of the displacement of the particle from its mean position.

(b) Kinetic Energy : This energy is on account of the velocity of the particle.

➢ **(a) Free oscillations :** A system capable of oscillating is said to be executing free oscillations if it vibrates with its own natural frequency without the help of any external periodic force.

The natural frequency depends upon inertia factors, and elastic properties & dimension of system.

(b) Forced Oscillations : When a body oscillates with the help of an external periodic force with a frequency different from the natural frequency of the body, its oscillations are called forced oscillations.

(c) Resonant Oscillations : When a body oscillates with its own natural frequency ν_0 with the help of an external periodic force whose frequency ν is equal to the natural frequency of the body ν_0, the oscillations of the body are called resonant oscillations.

- **Resonance :** It is a phenomenon of setting a body into vibrations with the help of another body vibrating with the same frequency.

or

The phenomenon of increase in amplitude when the frequency of the driving force is close to natural frequency of the oscillator is called resonance.

Know the Terms

- **Undamped Oscillations :** When a simple harmonic oscillator oscillates with a constant amplitude which does not change with time, its oscillations are undamped S.H.M.
- **Damped Oscillations :** When a simple harmonic oscillator oscillates with a decreasing amplitude with time, its oscillations are called damped S.H.M.
- **Coupled Oscillations :** Those oscillations in which two or more oscillating bodies connected to each other oscillate together & affect each other oscillations.

Know the Formulae

- **Potential Energy, S.H.M.** $U = \frac{1}{2}mw^2y^2 = \frac{1}{2}ky^2$
- **Kinetic Energy, S.H.M.** $K = \frac{1}{2}mw^2(a^2 - y^2)$

$$= \frac{1}{2}k(a^2 - y^2)$$

- **Total Energy, S.H.M.** $E = \frac{1}{2}mw^2a^2 = \frac{1}{2}ka^2$

Unit : w = Hz or s^{-1}

U, K, E = J.

❑❑

CHAPTER-15
WAVES

TOPIC-1
Waves & Wave Motion

Revision Notes

- **Wave motion** is a kind of disturbance which travels through a medium on account of repeated periodic vibrations of the particles of the medium about their mean position.
- **The medium** for wave propagation should have three properties :
  (a) elasticity
  (b) inertia
  (c) minimum frictional resistance.
- **Kinds of waves :**

(a) On the basis of necessity of material medium :

(i) Mechanical waves or elastic waves like sound waves, waves on the surface of water, waves on strings. All these waves require a material medium for propagation.

(ii) Electromagnetic waves like light waves, radio waves which require no medium for propagation.

(b) On the basis of vibrations of particles :

(i) Longitudinal waves : in which particles vibrate in the direction of propagation of waves.

(ii) Transverse waves : in which particles vibrate in a direction perpendicular to the direction of propagation of waves.

(c) On the basis of energy propagation :

(i) Progressive waves— in which energy is propagated.

(ii) Stationary waves—in which energy is confined in a particular region.

- **Waves :**

(a) Longitudinal waves travel through a medium in the form of compressions and rarefactions involving changes in pressure and volume and can travel in all modes and cannot be polarised. The medium required must possess elasticity of volume. Sound waves in air are longitudinal.

(b) Transverse waves travel through a medium in the form of crests and troughs involving changes in shape can travel in solid and liquid and can be polarised. The medium required must possess elasticity of shape. Vibrations in strings are transverse.

- **Laplace correction :** According to laplace, the changes in pressure & volume of a gas, when sound waves propagate through it are not isothermal but it is adiabatic. Because of :

(a) Velocity of sound in gas is quite large.

(b) A gas is bad conductor of heat.

$\therefore$ Velocity of sound, $v = \sqrt{\frac{B_a}{\rho}}$

B_a = Bulk modulus = γP, ρ = density of gas

Know the Terms

- A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium, when a transverse wave passes through it.
- A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when a transverse wave passes through it.
- A compression is a region of the medium in which particles are compressed *i.e.,* particles come closer or distance between particles become less than the normal distance between them. Thus, there is a temporary decrease in volume and a consequent increase in density of the medium in the region of compression.
- A rarefaction is a region of the medium in which *i.e.,* particles get farther apart than what they normally are. Thus there is a temporary increase in volume and a consequent decrease in density of the medium in the region of rarefaction.
- **Some parameters related to wave motion**

(a) Displacement of a particle is the distance covered by the particle from the mean position.

(b) Amplitude is the maximum displacement of the particle from the equilibrium position.

(c) One wavelength is the distance travelled by the wave, during the time the particle completes one vibration about its mean position. We may also define, one wavelength = smallest distance between two particles vibrating in the same phase = distance between the centres of two consecutive crests/troughs/compressions/rarefactions.

(d) Angular wave number or propagation constant : It is 2π times the no. of waves that can be accommodated per unit length. It is denoted by K.

(e) Frequency : It is the no. of complete wavelengths traversed by the wave in one second.

(f) Time period : It is equal to time taken by wave to travel a distance equal to wavelength.

(g) Particle velocity = velocity of particle executing SHM = $\frac{dx}{dt}$. Its value changes with time.

(h) Wave velocity is the velocity with which disturbance travels in the medium.

$$v = n\lambda = \frac{\lambda}{T} = \text{constant for a wave motion.}$$

(i) Group velocity (v_g) is the velocity with which the group of waves travel.

$$v_g = \frac{d\omega}{dK}$$

- **Some parameters related to sound waves :**

(i) Audible range : This frequency range lies between 20 Hz to 20,000 Hz. It is sensible to human ear.

(ii) Ultrasonic range : Any vibration whose frequency is greater than 20,000 Hz.

(iii) Infrasonic range : Sound waves which have frequencies less than the audible range are called infrasonic waves.

- **Phase or phase angle** is the physical quantity which tells us by what amount the two waves or the two particles lag or lead in terms of angle or time or distance.

- **Energy density** is defined as the energy associated with unit volume of the medium, *i.e.*,

  Energy density (u) = Energy/Volume.

  It is measured in $\frac{\text{Joule}}{\text{m}^3}$.

Know the Formulae

- **A longitudinal wave** can be represented by

$$x = a\sin(\omega t \pm kx)$$

A transverse wave can be represented by

$$y = a\sin(\omega t \pm kz)\ ;\ y = a\sin(\omega t \pm kx)$$
$$z = a\sin(\omega t \pm kx)\ ;\ z = a\sin(\omega t \pm ky)$$
$$x = a\sin(\omega t \pm ky)\ ;\ x = a\sin(\omega t \pm kz)$$

- **Harmonic wave function :** $y(x, t) = r\sin\left[\frac{2\pi}{\lambda}(vt - x) + \phi_o\right]$

  or $y(x, t) = r\cos\left[\frac{2\pi}{\lambda}(vt - x) + \phi_o\right]$

  where ϕ_0 = Initial phase.

- **Relation between phase difference, path difference and time difference**

  A phase difference of 2π radian is equivalent to a path difference of λ and a time difference of time period T, *i.e.*, $2\pi = \lambda$

  So, Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ Path difference

  Also, $\frac{x}{\lambda} = \frac{t}{T} = \frac{\phi}{2\pi}$

  where T is time period and it is time for a path x or phase ϕ.

- **Relation between particle velocity & wave velocity.**

$$u(x, t) = -v\frac{d}{dx}[y(x, t)]$$

  Where u = particle velocity, v = wave velocity.

- **Particle Acceleration :** $a(x, t) = -(2\pi\nu)^2 y = -\omega^2 y$.

- **Plane progressive wave :**

(a) Standard Equations : $y = r\sin\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$

or $y = r\cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right]$

where y = displacement, r = amplitude, T= time period, λ = wavelength, x = starting distance of wave from origin.

(b) Angular frequency : $\omega = \frac{2\pi}{T}$

(c) Propagation constant : $k = \frac{2\pi}{\lambda}$

(d) Velocity of wave : $v = \nu\lambda = \frac{\lambda}{T}$

(e) Velocity of particle : $u = \frac{dy}{dt}, u_{max} = r\omega$

(f) Acceleration of particle : $a = \frac{d^2y}{dt^2}, a_{max} = -\omega^2 r$

(g) Phase/path difference two waves :

(a) For two waves : $y_1 = a \sin \omega t,\ y_2 = b \cos \omega t$

Phase Difference : $(\omega t + \frac{\pi}{2} - \omega t) = \frac{\pi}{2}$

(b) For two waves : $y_1 = 10^{-6} \sin\left[100t + \frac{x}{50} + 0.5\right]$

$$y_2 = 10^{-6} \cos\left[100t + \frac{x}{50}\right]$$

Path Difference = 0.5 *radian.*

➢ **Newton's corrected formula for velocity of sound :**

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

TOPIC-2
Superposition Principle & Doppler Effect

Revision Notes

➢ **Principle of Superposition of Waves :**

(a) According to this principle, overlapping waves add algebraically to produce a resultant wave or a net wave.

or

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of displacements due to each wave at that time.

i.e., $y = y_1 + y_2 + \ldots\ldots\ldots + y_n$

Applications of Superposition Principle

(i) Stationary waves

(ii) Beats

(iii) Interference of waves.

➢ **Laws of Vibrations of Strecthed Strings.**

Fundamental Frequency of vibration of strecthed string

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

(a) Law of Length : $\nu \propto \frac{1}{L}$

Fundamental frequency is inversely proportional to length.

(b) Law of Tension : Fundamental frequency is directly proportional to square root of tension. *i.e.*

$$\nu \propto \sqrt{T}$$

(c) Law of Mass : Fundamental frequency is inversely proportional to the square root of mass :

$$\nu \propto \frac{1}{\sqrt{m}}$$

(d) Law of Diameter : $\nu \propto \frac{1}{D}$

(e) Law of Density : $\nu \propto \frac{1}{\sqrt{\rho}}$

➢ **Doppler Effect :** According to Doppler effect, wherever there is a relative motion between a source of sound & listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by source.

Doppler effect is related to change in frequency of sound.

Know the Terms

- **Stationary Waves :** If two waves of same type having same amplitude, same frequency and same wavelength, travelling with same speed in opposite directions along a straight line superimpose each other, they give rise to a new kind of waves known as stationary wave.
- **Longitudinal stationary waves** are formed as a result of super-imposition of two identical longitudinal waves travelling in opposite directions.
- **Transverse stationary waves** are formed as a result of super-imposition of two identical transverse waves travelling in opposite direction.
- **Anti-nodes** are certain points in the medium in a standing wave, the amplitude of vibration of which is maximum. Distance between two anti-nodes is $\lambda/2$.
- **Nodes** are certain points in the medium in a standing wave which are permanently at rest.
  Distance between two consecutive nodes is $\lambda/2$.
- **Beats** is the phenomenon of regular variation in the intensity of sound when two sources of nearly equal frequencies are sounded together.
  One minima of sound followed by one maxima is said to constitute one beat.
  The essential conditions for production of beats are :

(i) The amplitudes of two waves should be nearly equal.
(ii) The difference in frequencies of two sources must be less than 10 per second.

- **Beat period** is the time interval between two successive beats.
- **Beat frequency** is the number of beats produced per second.

Know the Formulae

- **Equation of stationary wave :** $y = 2r \sin \dfrac{2\pi x}{\lambda} \cos \dfrac{2\pi vt}{\lambda}$.
- **Normal modes of vibration of strings :**

(a) Fundamental frequency : $\nu_1 = \dfrac{1v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}} = \dfrac{1}{LD}\sqrt{\dfrac{T}{\pi\rho}}$

ν = frequency, L = resonance length, D = Diameter, ρ = Density, T = Tension.

(b) I overtone or 2nd harmonic $\nu_2 = 2\nu_1$

(c) II overtone or 3rd harmonic $\nu_3 = 3\nu_1$ & so on.

- **Organ Pipes :**

(a) Closed organ pipe :

(i) Fundamental note $\nu_1 = \dfrac{v}{4L}$

(ii) Ist overtone or IIIrd harmonic $\nu_2 = 3\nu_1$

(iii) IInd overtone or Vth harmonic $\nu_3 = 5\nu_1$ & so on

(b) Open organ pipe :

(i) Fundamental note $\nu_1 = \dfrac{v}{2L}$

(ii) Ist overtone or IInd harmonic $\nu_2 = 2\nu_1$

(iii) IInd overtone or IIIrd harmonic $\nu_3 = 3\nu_1$ & so on

- **Beats :** Beat frequency $m = n_1 - n_2$ or $n_2 - n_1$
  $n_2 = n_1 \pm m$.
- **Doppler's Effect**

$$\nu' = \frac{\{(v + v_m) - v_L\}\nu}{(v + v_m) - v_s}$$

ν' = Apparent frequency of sound heard
ν = Actual frequency of sound
v_m = Velocity of medium (air)
v_s = Velocity of source
v_L = Velocity of listener.